Chapter 3 Answers

These are answers to the exercises in Chapter 3 of:

Understanding the Properties of Matter
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

Thanks

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P1.

(a) Magnetic flux density (tesla, T)

(b) Temperature (kelvin (K) or degrees celsius (°C))

(c) Electrical conductance (seimen, S)

(d) Electrical resistance (ohm, Ω)

(e) Electrical capacitance (farad, F)

(f) Amount of substance (mole, mol)

(g) Mass (kilogram, kg)

P2.

(a) 1 mole of carbon atoms, 12.01×10^{-3} kg

(b) 1 mole of nitrogen atoms, 14.01×10^{-3} kg

(c) 1 mole of nitrogen molecules, 28.02×10^{-3} kg

P3. The volume of a kilogram of platinum iridium is given by:

$$\begin{aligned} volume &= \frac{mass}{density} \\ &= \frac{1}{22000} \\ &= 4.54 \times 10^{-5} \ \text{m}^3 \end{aligned}$$

If the kilogram is in the form of a cube it will have a side L given by:

$$\begin{aligned} L &= \sqrt[3]{4.54 \times 10^{-5}} \ \text{m} \\ &= 0.0357 \text{ m} \\ &= 35.7 \text{ mm} \end{aligned}$$

The surface area of the kilogram would be roughly $6L^2$ and so the volume of a layer of water of thickness t is $6tL^2$. Water has a density of 1000 kg m^{-3} and so a microgram of water would have a volume given by:

$$\begin{aligned} volume &= \frac{mass}{density} \\ 6tL^2 &= \frac{10^{-9}}{1000} \\ t &= \frac{10^{-9}}{6 \times [0.0357]^2 \times 1000} \\ &= 0.13 \ \text{nm} \end{aligned}$$

This is of the order of a layer a single molecule thick. Its pretty clear that the uncertainty in the mass of a kilogram could never be reduced below a microgram.

P4. If the uncertainty is 1 part in 10^{13} then in the worst case (i.e. assuming that the uncertainties conceal a systematic drift of the frequency in one direction) it will take 10^{13} seconds for the time shown by the clock to become uncertain by 1 second. 10^{13} seconds amounts to:

$$\begin{aligned}
\text{1 second in } & \frac{10^{13}}{60}\text{ minutes} \\
&= \frac{10^{13}}{60\times 60}\text{ hours} \\
&= \frac{10^{13}}{60\times 60\times 24}\text{ days} \\
&= \frac{10^{13}}{60\times 60\times 24\times 365.25} \\
&\approx 3\times 10^{5}\text{ years}
\end{aligned}$$

With a similar assumption, that the time becomes uncertain by 0.1 picoseconds every second, after 100 years i.e. after:

$$\begin{aligned}
&\approx 100\times 365\times 24\times 60\times 60 \\
&= 3.156\times 10^{9}\ \text{ seconds}
\end{aligned}$$

the uncertainty will be approximately $3.156\times 10^{9}\times 10^{-13}\approx 3.1\times 10^{-4}$ seconds or 0.31 ms.

Although these answers indicate that the clocks are accurate for *many* practical purposes, they are still not good enough for *all* practical purposes. For example, to improve the accuracy of the *Global Positioning System* beyond its current accuracy (≈10 m) will require still more accurate clocks. (See *Scientific American* February 1996 Vol 274 No.2 *The Global Positioning System* by Thomas A. Herring)

P5. We proceed as follows:

- There are 31.5×10^{6} seconds/year (≈ 30 million) so 1 cm per year = 3.17×10^{-10} ms^{-1}
- 1 cm is ≈ $10^{-2}/(0.3\times 10^{-9}\text{m})$ ≈ 33 million rows of atoms per second.
- My hair grows as about 2 or 3 centimetres each month which is about 30 times faster than the rate at which the USA and Europe are diverging.

The measurement requires a resolution of the order of 1mm in more than 1000 km i.e. a resolution of better than 1 in 10^{6}. The global positioning system does not yet have this resolution. The problem with any kind of interferometry is that the conditions of the atmosphere change the refractive index of radio waves or light waves by parts in 10^{6} and so an interference experiment between, say, radio waves sent down a transatlantic cable (assumed to be a relatively stable reference path) and across the ocean through the atmosphere would show large fluctuations with the weather which would be hard to unambiguously compensate for. It would take many decades to remove the effect of seasonal fluctuations. However, a technique known as Very Long Baseline Interferometry (VLBI) does allow such determinations to be made. If I understand it correctly (!) observations are made at two widely separated sites (e.g. USA and Europe) of very distant objects that emit very precisely timed pulses of radio waves. After much averaging and careful corrections, comparing precise timings of measurements made at the two different sites, it is possible to work out their separation. Any measurement would take several years to detect as systematic shift in the positions of the continents.

I believe the way these measurements were originally made was by examining the magnetisation of the "new" material at the mid-Atlantic ridge. Over a time-scale of a few tens of thousands of years, the earth's magnetic field has reversed and the direction magnetisation of the rocks is characteristic of the earth's field *when the rock became solid*. By comparing these bands with other evidence for the timing of geomagnetic reversals, the separation speed of the USA and Europe is inferred.

P6. First we split the integral into two part corresponding to the two types of wire:

$$V_{AC} = \int_A^B S(x)\frac{dT}{dx}\mathrm{d}x + \int_B^C S(x)\frac{dT}{dx}\mathrm{d}x$$

Then we note that within each type of wire the Seebeck coefficient is a constant and can be taken outside the integral. (Notice that if the composition of the wires changes with position then this step cannot be taken and thermocouples would not very valuable. The maintenance of a constant alloy composition along the length of reel of thermocouple wire is a major technical problem in the high temperature use of thermocouples.)

$$V_{AC} = S_1\int_A^B \frac{dT}{dx}\mathrm{d}x + S_2\int_B^C \frac{dT}{dx}\mathrm{d}x$$

We can now simplify the integral as follows:

$$V_{AC} = S_1\int_A^B \frac{dT}{dx}\mathrm{d}x + S_2\int_B^C \frac{dT}{dx}\mathrm{d}x$$

$$V_{AC} = S_1\int_A^B dT + S_2\int_B^C dT$$

$$V_{AC} = S_1\,[T]_A^B + S_2[T]_B^C$$

$$V_{AC} = S_1\left[T_B - T_A\right] + S_2\left[T_C - T_B\right]$$

Now we need to note that:

$$T_A = T_R$$

$$T_B = T_J$$

$$T_C = T_R$$

And after substituting we arrive at:

$$V_{AC} = S_1\left[T_J - T_R\right] + S_2\left[T_R - T_J\right]$$

$$V_{AC} = \left[T_R - T_J\right]\times\left[S_2 - S_1\right]$$

Given the physical origin of the effect, it is astounding that the temperature at "the junction" at which precisely no voltage is generated essentially defines the measured voltage!

P7. (a) A photo resistor might easily react within a few milliseconds so it is not exactly slow, but that's another story. Possible applications might be to monitor ambient light level and switch on a light when the ambient light level is too low.

(b) A photodiode might be used to detect the fast pulses of light travelling down an optical fibre (at a MHz or so), or perhaps in a safety critical sensor that could rapidly (i.e. within a single mains cycle) cut power to a motor when a light beam was blocked..

P8. Green light: single photon

$$
\begin{aligned}
E &= hf \\
&= \frac{hc}{\lambda} \\
&= \frac{6.62\times10^{-34}\times3\times10^{8}}{532\times10^{-9}} \\
&= 3.73\times10^{-19}\ \text{ J}
\end{aligned}
$$

Number of photons per seconds

$$
\begin{aligned}
&= \frac{10^{-10}\ \text{ W}}{3.73\times10^{-19}\ \text{ J}} \\
&= 2.67\times10^{8}\ \text{ photon s}^{-1}
\end{aligned}
$$

If 1 electron per photon

$$
\begin{aligned}
i &= 2.67\times10^{8}\times\left[1.6\times10^{-19}\right] \\
&= 4.47\times10^{-11}\ \text{ A}
\end{aligned}
$$

If I used a photomultiplier (with a gain of perhaps 10^5) I would get just under a micro-amp which would be rather easier to measure. However, in the photomultiplier there is a short time (of the order of a microsecond) after each pulse when the sensitivity of the photo-multiplier is reduced. If a photon arrives at the photomultiplier during this time it will not generate as much current as it would if it arrived independently. The photomultiplier would be producing 2.67×10^8 pulses per second and so many of the pulses would fall in this so-called dead time. This introduces a non-linearity into the response of the photomultiplier which is something I am very keen to avoid.

P8. We approach this problem with the following simplifications:

- 1 kW m^{-2} of optical energy strikes the Moon's surface
- All the light that strikes the moon's disc is re-radiated so the radiated power is given by

$$
\begin{aligned}
\textit{Radiated Power} &\approx \left[\text{ Incident Intensity}\right]\times \pi R^2_{\text{Moon}} \\
&= 10^3 \times \pi\left[\frac{3.5\times 10^6}{2}\right] \\
&\approx 9.621\times 10^{15}\ \text{W}
\end{aligned}
$$

- This re-radiation is in all directions (except through the moon!) So the power is distributed across the surface of a hemisphere. So the radiant intensity at the earth is:

$$
\begin{aligned}
&\approx \frac{\textit{Total Energy Radiated}}{\textit{Surface area of hemisphere}} \\
&\approx \frac{9.621\times 10^{15}\ \text{W}}{2\pi R^2_{\text{Moon-Earth}}} \\
&\approx \frac{9.621\times 10^{15}}{2\pi\left[384\times 10^6\right]^2} \\
&\approx\ 0.01\ \text{Wm}^{-2}
\end{aligned}
$$

i.e. ≈ 1 part in 10^5 of the radiant flux during the day. Does that seem about right for a bright moonlit night? I think so.

- Assume the light is all the same colour (frequency) ≈ 7 × 10^{14} Hz so the energy of a single photon is $hf \approx 4.4 \times 10^{-19}$ J so there are:

$$
\begin{aligned}
&\approx \frac{0.01\ \ \text{W m}^{-2}}{4.4\times 10^{-19}\ \ \text{J photon}^{-1}} \\
&= 2.4\times 10^{16}\,\text{photon m}^{-2}\ \text{s}^{-1}
\end{aligned}
$$

- So ≈ 2 × 10^{16} photons are striking each square metre of the Earth per second. In fact, most of this radiation striking the Moon in the first place is not optical but infra-red. However even allowing for several factors of 10 errors, there is still a considerable flux of optical photons.

P10. We approach this problem with the following simplifications:

- We assume the Sun is a disc of radius given by R_{Sun}. It therefore has an emitting area of πR^2_{Sun}. (Astute readers may argues that the sun is spherical and that emits radiation from a greater surface area. However the "extra area" is not directly facing the Earth and the calculation then has a sinusoidal factor which when integrated over a hemisphere exactly compensates for the extra area)
- The total energy emitted by the Sun's disc in our general direction is then

$$P = A\varepsilon\sigma T^4$$
$$= \pi R_{\text{Sun}}^2 \varepsilon\sigma T^4$$

- We assume that the energy emitted above is uniformly distributed over the surface of a hemisphere with radius equal to the Sun-Earth separation. Thus the measured intensity of solar radiation at the Earth's surface (1 kW m^{-2}) should be equal to:

$$10^3 = \frac{\pi R_{\text{Sun}}^2 \varepsilon\sigma T^4}{2\pi R_{\text{Sun-Earth}}^2}$$

- Solving this for the surface temperature of the Sun assuming an emissivity of unity yields:

$$T^4 = \frac{10^3 \times 2 R_{\text{Sun-Earth}}^2}{\sigma R_{\text{Sun}}^2}$$
$$T = \sqrt[4]{\frac{10^3 \times 2 \times \left[150 \times 10^9\right]^2}{5.67 \times 10^{-8}\left[7 \times 10^8\right]^2}}$$
$$= 6343 \quad K$$

Concerning your body, we can guess that the emissivity of your skin is around 0.5, and that you have roughly 2 m^2 of skin on your body. We then find that the emitted power is:

$$P = 2 \times 0.5 \times 5.67 \times 10^{-8} \times [22 + 273]^4$$
$$\approx 430 \quad \text{W}$$

This is a considerable amount of energy.

P11. Using the fact the number of wavelengths in the tube $N = Ln_{\text{light}}/\lambda_o$ the fact that $n_{\text{light}} = 1 + P\alpha/2\varepsilon_o k_B T$ we find

$N = L\,[1 + P\alpha/2\varepsilon_o k_B T]/\ \lambda_o$

If we slowly change the pressure in the sample tube, the number of wavelengths will change, and the interference with the reference beam will go succesively from constructive to destructive, and then back to constructive, with one complete cycle corresponding to the introduction of one whole extra wavelength into the sample tube. A graph of Pversus N will have slope $\alpha/2\varepsilon_o k_B T\lambda_o$ from which α may be estimated

P12. The voltmeter is assumed to have a very large internal resistance so that one may assume that no current flows through it. In practice $10^8\ \Omega$ is common and so it is not a bad assumption for many purposes since at balance no current flows through it anyway.

The voltage change ΔV is less than the voltage change $i_d \Delta R_d$ across R_d and so it seems that some signal has been lost. However the key point is that in the bridge configuration, the voltmeter may be used on its most sensitive scale since ΔV is to be compared with $\Delta V = 0$ at balance. Whereas if the voltage across R_d was measured directly, then the voltage $i_d R_d$ will not in general fit on the most sensitive scale and the signal voltage $i_d \Delta R_d$ forms only a small fraction of the total voltage, $i_d R_d + i_d \Delta R_d$

P13. A capacitance change of 1 part in 10^8 translates directly into a separation change of 1 part in 10^8 of d = 0.1mm i.e. $\approx 10^{-4}/10^8 \approx 10^{-12}$. This corresponds to ≈ 1/300 of the diameter of a typical atom. Most amazingly of all, this sensitivity can be achieved in practice!

P14.

- Speed on hitting the taper is 4.43 ms^{-1} and the momentum of the weight is 44.3 $kgms^{-1}$.
- Estimate that the weight stops in 1 millisecond, then the force exerted transiently is ≈ 44.3 kg ms^{-1}/0.001 ≈ 44.3 kN.
- This force is transmitted to the base of the taper where it acts over an area of $\pi \times (10^{-4})^2$ m^2 and corresponds to a pressure of $\approx 44.3 \times 10^3/\pi \times 10^{-8} \approx 10^{12}$ Pa ≈ 10^7 atmospheres. i.e. a very large pressure.

High pressure apparatuses usually use hydraulic press to achieve a constant rather than a transient pressure, but otherwise the principle is the same. The problem with achieving such high pressures is that (as in this experiment) nearly everything breaks at this pressure!

P15.

$$
\begin{aligned}
P &= P_o \exp\left[-\frac{St}{V}\right] \\
\ln\left[\frac{P}{P_o}\right] &= -\frac{St}{V} \\
\ln\left[\frac{1}{10^5}\right] &= -\frac{[8/3600]t}{0.1} \\
t &= -\frac{0.1}{[8/3600]} \times \ln\left[\frac{1}{10^5}\right] \\
&= 518 \text{ seconds} \\
&\approx 8\tfrac{1}{2} \text{ minutes}
\end{aligned}
$$