

Chapter 4 Answers

These are answers to the exercises in Chapter 4 of:

Understanding the Properties of Matter
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

Thanks

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P1. $P \approx 9.74 \times 10^6$ Pa which is ≈ 97 times atmospheric pressure. Need to reduce temperature by a factor of roughly 97 i.e. $293/97 \approx 3$ K. All gases will have liquefied or solidified by this temperature and so it would not be necessary to lower the temperature quite this low.

P2. (a) $n \approx 2.5 \times 10^{25} \text{ m}^{-3}$ **(b)** $n \approx 2.5 \times 10^{25} \text{ m}^{-3}$

C3. On the web site see under the 'equations' tab for Chapter 4

P4. Using $v \approx 500 \text{ ms}^{-1}$ we find a collision frequency of $\approx 3.1 \times 10^{27} \text{ m}^{-2} \text{ s}^{-1}$

Atom is struck roughly $2782P$ times per second (P in Pa). To reduce this frequency to 1 collision in 1000 seconds requires $P \approx 3.6 \times 10^{-7}$ Pa which is an ultra high vacuum.

P5. $v_{\text{rms}} \approx 427 \text{ ms}^{-1}$, $\lambda_{\text{mfp}} \approx 0.1 \mu\text{m}$ $\Delta t \approx 0.23 \text{ ns}$ i.e. 4.27×10^9 collisions per second

P6. The experimental method by which the speed distribution curve is determined usually employs an oven with a pinhole which "samples" a small fraction of the gas within the oven. It is passed through a velocity selecting device and the number of molecules leaving with a particular speed estimated.

P7. (a) I think the simplest *riposte* is to point out the constancy of the molar volume (or density) at STP. This is pretty remarkable. **(b)** Cooling gases make them condense at some temperature or other, However $PV = zRT$ is obeyed until very close to the condensation temperature.

P8. Measurements are made of the speed of 20 molecules sampled from some argon gas with the following results:

101.3	308.5	451.7	500.8
126.3	357.5	468.6	503.2
152.0	379.2	478.1	527.9
174.3	379.9	482.1	574.4
304.5	417.0	492.1	867.0

(a) $\bar{v} = 402.3 \text{ ms}^{-1}$ and

$$v_{\text{RMS}} = 438.7 \text{ ms}^{-1}$$

Ratio ≈ 1.090 compared with expected value 1.086

Could I really have done an experiment to measure the speed of 20 individual molecules? Or did I just make up the numbers for this question?

(b) Knowing the mean speed and molecular mass we can estimate the average kinetic energy per molecule and then the temperature according to Equation 4.24

$$\begin{aligned}\frac{1}{2}m\overline{v^2} &= \frac{3}{2}k_{\text{B}}T \\ T &= \frac{m\overline{v^2}}{3k_{\text{B}}} \\ &= \frac{40 \times 1.66 \times 10^{-27} \times 438.7^2}{3 \times 1.38 \times 10^{-23}} \\ &= 308.7 \text{ K}\end{aligned}$$

C9. You should see a distinct change in the nature of the trajectories. When molecular interactions are switched on the interactions cause the trajectories to be very non-linear when pairs of molecules are close together. When molecular interactions are switched off, (i.e. the idea gas approximation) the trajectories are straight lines by definition.

P10. We use the formula for molar density $\frac{z}{V} = \frac{P}{RT}$

mass density = molar mass \times molar density

$$\rho = \frac{P}{RT} \times 18 \times 10^{-3} = 0.0074 \text{ kg m}^{-3}$$

50% of this is amounts to 0.0037 kg m^{-3} which is a significant contribution to the density of air, and goes some way to explaining the variations in the density of air at around room temperature.

P11. You should find something like 34.8 Pa for a three metre tall room at 20 °C. In general the result is $\Delta P \approx -11.6z \text{ Pa}$ (where z is in metres) as long as z is much small than the scale height of the atmosphere $\approx 8 \text{ km}$