

## **Chapter 5 Answers**

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These are answers to the exercises in Chapter 5 of:

*Understanding the Properties of Matter*  
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

Thanks

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**P1.** Only (d) and (e). This might help you if you have not already answered Exercise P8 in Chapter 4

**P2.** (a)  $22.4 \times 10^{-3} \text{ m}^3$  i.e. 22.4 litres (b)  $24.3 \times 10^{-3} \text{ m}^3$  i.e. 22.3 litres

My shared office is  $6\text{m} \times 4\text{m} \times 4\text{m} \approx 100 \text{ m}^3$

	N <sub>2</sub>	O <sub>2</sub>	<b>Air</b>
<b>Amount of Gas</b>	3285 mol	821 mol	<b>4106 mol</b>
<b>Mass</b>	92 kg	26 kg	<b>118 kg</b>
<b>Volume in the solid phase</b>	$\approx 0.088 \text{ m}^3$	$\approx 0.0178 \text{ m}^3$	<b><math>\approx 0.1 \text{ m}^3</math></b>

Note volume ratio of  $\approx 1000$  between solid and gas

**P3. Mass**  $1.69 \times 10^{-3} \text{ kg}$ ,  $0.182 \times 10^6 \text{ Pa}$

**P4.** Compare the experimental results on the pressure coefficient  $b_p$  (Table 5.4) with those of a theory of your own devising. What is the relationship between the deviations from theory of the pressure and volume coefficients (Table 5.5)?

**E5.** The excursions of the gas are shown in the figure

- (a)  $0.655 \times 10^5 \text{ Pa}$  ( $\approx 0.65 \text{ bar}$ )
- (b) 589 N
- (c)  $\approx 2166 \text{ K}$  ( $\approx 1890 \text{ }^\circ\text{C}$ )
- (d)  $0.495 \times 10^5 \text{ Pa}$  ( $\approx 0.495 \text{ bar}$ ), 1429 K (1156  $^\circ\text{C}$ )

**P6.** Overall densities equal implies:

$$\frac{\text{Load} + \text{Mass of He}}{\text{Balloon Volume}} = \rho_{\text{air}}(\text{STP})$$

$$\frac{\text{Load}}{\text{Balloon Volume}} + \rho_{\text{He}} = \rho_{\text{air}}(\text{STP})$$

$$\text{Balloon Volume} = \frac{\text{Load}}{\rho_{\text{air}}(\text{STP}) - \rho_{\text{He}}}$$

and using the average molar mass for air  $M_{\text{air}} = 28.8 \times 10^{-3}$  we find  $\rho_{\text{air}} \approx 1.286 \text{ kg m}^{-3}$  and using  $M_{\text{He}} = 4 \times 10^{-3}$  we find  $\rho_{\text{He}} \approx 0.194 \text{ kg m}^{-3}$ . This implies that...

$$\text{Balloon Volume} = \frac{\text{Load}}{1.286 - 0.194} \approx 0.916 \times \text{Load}$$

- (a) Load = 10 g Volume is  $\approx 9.16 \times 10^{-3} \text{ m}^3$  i.e. a sphere of diameter  $\approx 26 \text{ cm}$
- (b) Load = 70 kg Volume is  $\approx 64.1 \text{ m}^3$  i.e. a sphere of diameter  $\approx 5 \text{ m}$

If the temperature increases, then the density of the air will fall, but the density of the helium is fixed by the volume of the container. Thus the overall lift will be reduced, and so the balloon will sink.

**P7.** The uplift on a body is given by *Archimedes* principle

$$Uplift = [\text{Volume of object or weight}] \times [\text{density of displaced air}] \times g$$

So for the object being weighed and the weights we have:

$$\text{Uplift on object} = g \left[ \frac{m_{\text{object}}}{\rho_{\text{object}}} \right] \times \rho_{\text{air}} \qquad \text{Uplift on weights} = g \left[ \frac{m_{\text{weights}}}{\rho_{\text{weights}}} \right] \times \rho_{\text{air}}$$

In vacuum (no uplift) balance is achieved when:

$$m_{\text{object}}g = m_{\text{weights}}g \quad \text{i.e.} \\ m_{\text{object}} = m_{\text{weights}}$$

In air, balance is achieved when:

$$m_{\text{object}}g - [\text{Uplift on object}] = m_{\text{weights}}g - [\text{Uplift on weights}] \\ m_{\text{object}}g - g \left[ \frac{m_{\text{object}}}{\rho_{\text{object}}} \right] \times \rho_{\text{air}} = m_{\text{weights}}g - g \left[ \frac{m_{\text{weights}}}{\rho_{\text{weights}}} \right] \times \rho_{\text{air}} \\ m_{\text{object}} - \left[ \frac{m_{\text{object}}}{\rho_{\text{object}}} \right] \times \rho_{\text{air}} = m_{\text{weights}} - \left[ \frac{m_{\text{weights}}}{\rho_{\text{weights}}} \right] \times \rho_{\text{air}}$$

i.e. when:

$$m_{\text{weights}} = m_{\text{object}} + \rho_{\text{air}} \left[ \left[ \frac{m_{\text{weights}}}{\rho_{\text{weights}}} \right] - \left[ \frac{m_{\text{object}}}{\rho_{\text{object}}} \right] \right]$$

which in general does not give the correct answer! If  $m_{\text{weights}} \approx m_{\text{object}}$  we can rewrite this as:

$$\frac{m_{\text{weights}}}{m_{\text{object}}} = 1 + \rho_{\text{air}} \left[ \left[ \frac{1}{\rho_{\text{weights}}} \right] - \left[ \frac{1}{\rho_{\text{object}}} \right] \right]$$

which for the instance mentioned evaluates to:

$$\frac{m_{\text{weights}}}{m_{\text{object}}} = 1 + 1.2 \left[ \left[ \frac{1}{8000} \right] - \left[ \frac{1}{2000} \right] \right] \\ = 1 - 4.5 \times 10^{-4}$$

i.e. its mass will be underestimated by the balance. In most cases the results deviate from unity by of the order of 1 part in  $10^4$ .

**P8.** According to this analysis, the volume of atmosphere is

$$\begin{aligned}V &= 4\pi R_{\text{earth}}^2 \times [\text{height of atmosphere}] \\&= 4\pi [6.4 \times 10^6]^2 \times [10^4] \\&= 5.15 \times 10^{18} \text{ m}^3\end{aligned}$$

So the number of moles in the atmosphere is

$$\begin{aligned}z &= \frac{PV}{RT} \\&= 4\pi [6.4 \times 10^6]^2 \times [10^4] \\&= \frac{0.5 \times 10^5 \times 5.15 \times 10^{18}}{8.31 \times 273} \\&= 1.13 \times 10^{20} \text{ mol}\end{aligned}$$

Since 1 mol has a mass of roughly  $28.8 \times 10^{-3}$  kg we find that the mass is

$$\begin{aligned}z &= [1.13 \times 10^{20}] \times [28.8 \times 10^{-3}] \\&= 3.27 \times 10^{18} \text{ kg}\end{aligned}$$

Similarly we find the mass of  $\text{CO}_2$  in atmosphere to be roughly  $1.49 \times 10^{15}$  kg

So there is roughly  $4.08 \times 10^{14}$  kg of C in atmosphere

So for 1% increase per year  $4.08 \times 10^{12}$  kg year<sup>-1</sup> or around 4 billion tonnes year<sup>-1</sup>

**P9.** Looking at the tables we find:

(a)  $20.8 \text{ J K}^{-1} \text{ mol}^{-1}$ ,

(b)  $29.1 \text{ J K}^{-1} \text{ mol}^{-1}$ ,

(c)  $34.9 \text{ J K}^{-1} \text{ mol}^{-1}$ ,

**P10.** Looking at the tables we find:

(a) Nitrogen  $29.1 \text{ J K}^{-1} \text{ mol}^{-1}$ ,

(b) Oxygen  $29.5 \text{ J K}^{-1} \text{ mol}^{-1}$ ,

(c) Argon  $20.786 \text{ J K}^{-1} \text{ mol}^{-1}$

**P11.** From Table 5.8 we find:

(a) $\gamma = 1.400$	$p = 5.00$	oxygen
(b) $\gamma = 1.401$	$p = 4.99$	nitrogen
(c) $\gamma = 1.667$	$p = 3.00$	argon
(d) $\gamma = 1.30$	$p = 6.67$	carbon dioxide
(e) $\gamma = 1.401$	$p = 4.99$	air

It is often a fair approximation to treat the thermal properties of air as being similar to nitrogen, but one should use an effective molar mass of 28.8 g ( $=0.8 \times 28 + 0.2 \times 32$ ) to take account of the presence of oxygen.

**P12.**

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	He	Hg	Kr	Air	H <sub>2</sub> O	CH <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>
$\gamma$	1.63	1.666	1.689	1.357	1.334	1.313	1.22
p	3.17	3.00	2.90	5.60	5.99	6.39	9.09

Rather difficult to describe.

**P13.**

(a)  $\approx 521 \text{ J K}^{-1} \text{ kg}^{-1}$  for argon approximately and  $\approx 5200 \text{ J K}^{-1} \text{ kg}^{-1}$  for helium

(b)  $\approx 927 \text{ J K}^{-1} \text{ m}^{-3}$  for argon and the same for helium and all near-perfect gases

**P14.** (Note: I wrote this question at home and not in my office as in exercise P2)

$$z = \frac{PV}{RT}$$

$$\approx \frac{10^5 \times [3 \times 4 \times 5]}{8.31 \times 290}$$

$$\approx 2490 \text{ mol}$$

$$\text{heat capacity} = 2490 \times 29.1$$

$$\approx 72451 \text{ J K}^{-1}$$

$$\text{time} = \frac{\text{heat capacity} \times \text{temperature rise}}{\text{power}}$$

$$= \frac{72451 \text{ J K}^{-1} \times [25 - 8]}{1000 \text{ J s}^{-1}}$$

$$\approx 1232 \text{ s}$$

or  $\approx$  roughly 20 minutes. Where is most of the heat stored. It takes a while to calculate, but I'm pretty sure that its nearly all stored in the air rather than the wall. You can see this because of the rate at which the room warms: essentially you should find that there is no "spare heat" to heat the walls.

**P15.**

	H <sub>2</sub>	Cl <sub>2</sub>	O <sub>2</sub>
Key Temperature	3000 K	750 K	2000 K
resonant frequency $f_0$		$1.57 \times 10^{13}$ Hz	$4.2 \times 10^{13}$ Hz
Spring constant $K$	$\approx 258 \text{ Nm}^{-1}$	$\approx 573 \text{ Nm}^{-1}$	$\approx 1849 \text{ Nm}^{-1}$

not calculated using the reduced mass

**P16.**

$$\text{Number of moles} \approx 1.13 \times 10^{20} \text{ mol}$$

$$\text{Heat Capacity} \approx 1.13 \times 10^{20} \times 29.1$$

$$\approx 3.29 \times 10^{21} \text{ J K}^{-1}$$

i.e. It takes  $3.29 \times 10^{21}$  joules to raise the atmospheric temperature by 1 K. Roughly speaking there is 1 kW of thermal energy striking every square metre of the Earth from the Sun which amounts to a heat input per second of very roughly  $10^3 \times [\pi R_{\text{earth}}^2] = 1.28 \times 10^{17}$  joules. Over one year this amounts to roughly 30 million times this amount.  $\approx 3.86 \times 10^{24}$  joules. The actual temperature of the Earth is the result of complex balance between solar radiation input and the amount of radiation re-radiated from Earth into space. The aim of this question is just to give you some kind of idea of the orders of magnitude of the quantities involved.

**P17.** We start with:

$$PV^\gamma = K_1$$

where  $K_1$  is a constant. We now substitute for  $P$  and simplify the result

$$\frac{P}{zRT} V^\gamma = K_1$$

$$\frac{zRT}{V} V^\gamma = K_1$$

$$TV^{\gamma-1} = \frac{K_1}{zR}$$

In any particular expansion  $zR$  is also a constant so we can write:

$$TV^{\gamma-1} = K_2$$

where  $K_2$  is a constant.

**C18.** See web site for animated example under Chapter 5: Equations

**P19.** Comparing the situations before (i) and after (f) the compression we have:

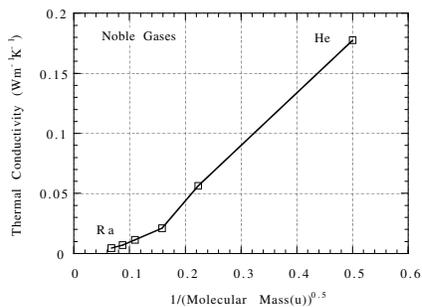
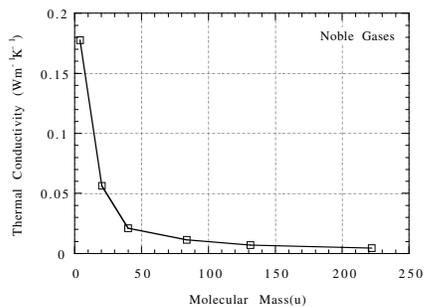
$$\begin{aligned}
 T_f V_f^{\gamma-1} &= T_i V_i^{\gamma-1} \\
 T_f &= \frac{T_i V_i^{\gamma-1}}{V_f^{\gamma-1}} \\
 &= T_i \left[ \frac{V_i}{V_f} \right]^{\gamma-1}
 \end{aligned}$$

substituting the numbers from our problem we arrive at:

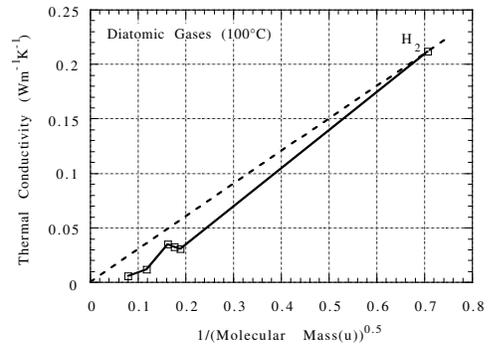
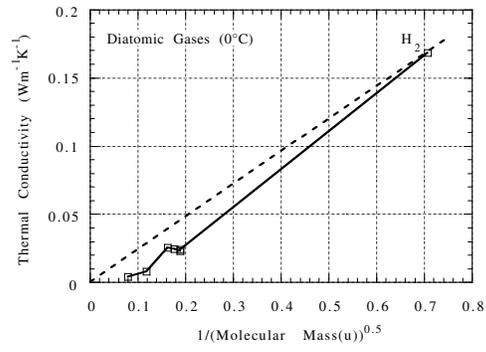
$$\begin{aligned}
 T_f &= 290 \left[ \frac{10}{2.5} \right]^{1.4-1} \\
 &= 290 \times 1.74 \\
 &= 504.9 \text{ K} \\
 &\approx 232 \text{ celsius}
 \end{aligned}$$

Its difficult to determine experimentally whether this is realistic but the valve on a bicycle does get very hot. Bicycle tyre pressure very enormously from about 2 atmospheres ( $2 \times 10^5$  Pa) to around 5 atmospheres ( $5 \times 10^5$  Pa) for a racing bike. Based on the tyre pressure you achieve, you can determine the calculation for your own bicycle .

**C20.**



What this brings out is that a straight line is not obtained because the effective molecular diameter is differs between the molecules. Scaling all the values up to the “expected value” based a line through the origin to He would reveal the factor by which  $a^2$  is larger for each molecule than it is for He. The data for the diatomic gases show these effects even more strongly than do the monatomic gases.



C21. Do it!

**P22.** Assuming a temperature around room temperature,  $\lambda_{\text{mfp}} \approx 77 \text{ mm}$ .

Use  $\lambda_{\text{mfp}} = \frac{1}{\sqrt{2}n\pi a^2}$  and substitute for  $n$  using  $n = \frac{P}{k_{\text{B}}T}$  to find

$$\begin{aligned}\lambda_{\text{mfp}} &= \frac{1}{\sqrt{2} \frac{P}{k_{\text{B}}T} \pi a^2} \\ &= \frac{k_{\text{B}}T}{\sqrt{2}P\pi a^2} \\ &= \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 1 \times \pi \times [0.108 \times 10^{-9}]^2} \\ &= 0.077 \text{ m}\end{aligned}$$

The thermal conductivity will be well below that of the plateau region since  $\lambda_{\text{mfp}}$  is very much greater than the dimension of the container ( $\approx 1 \text{ mm}$ ).

**P23.** Assuming a filament temperature of  $800 \text{ }^\circ\text{C}$ , a pressure of  $\approx 100 \text{ Pa}$  should yield a mean free path of the order of the distance from the filament to the cold walls of the container. This yields an estimate  $\lambda_{\text{mfp}} \approx 0.03 \text{ mm}$ . In practice devices are rather larger than this ( $\approx 1 \text{ mm}$ ) and sensitive at rather lower pressures ( $\approx 1 \text{ Pa}$  to  $10 \text{ Pa}$ )

**P24.**

$$(a) \ z = \frac{PV}{RT} = \frac{10^5 \times 1}{8.31 \times 293} \approx 41 \text{ mol}$$

(b)  $0.1 \text{ mol}$  of gas transports excess heat energy of

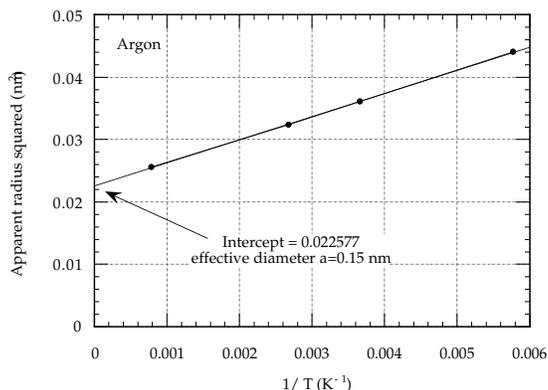
$$\begin{aligned}zC_p\Delta T &\approx 0.1 \times 29 \times 10 \\ &\approx 29 \text{ J s}^{-1}(\text{W})\end{aligned}$$

So convection transports  $\approx 29 \text{ W}$

The still gas heat flow is given by

$$\begin{aligned}\text{Heat flow} &= \kappa A \frac{\Delta T}{\Delta x} \\ &\approx 2.5 \times 10^{-2} \times 1 \times \frac{10}{1} \\ &\approx 0.25 \text{ W}\end{aligned}$$

C25. The Sutherland plot for argon is given by:

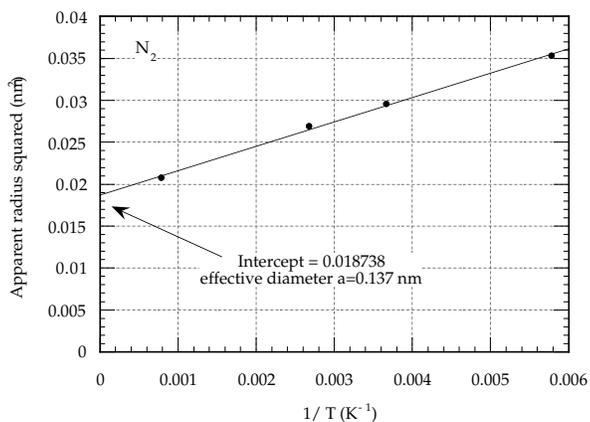


Analogous to Example 5.9 we predict that

T (K)	173	273	373	1273
$\kappa(\text{experiment})$	$1.59 \times 10^{-2}$	$2.40 \times 10^{-2}$	$3.09 \times 10^{-2}$	$7.40 \times 10^{-2}$
$\kappa(\text{expect})$	$6.26 \times 10^{-3}$	$7.857 \times 10^{-3}$	$9.184 \times 10^{-3}$	$16.96 \times 10^{-3}$
ratio	2.541	3.05	3.365	4.362
N <sub>2</sub> "diameter"	0.188	0.172	0.164	0.144

And the equivalent Sutherland plot is shown below:

The Sutherland plot is not quite as straight as for Argon, but then one wouldn't really expect it to be. At high temperatures the vibration of the molecule may well serve to effectively enlarge the molecule a little bit. The effective diameter probably refers to the "swept area of the molecule" i.e. the area swept out by a rotating dumbbell. Basically though, the monatomic analysis is confirmed.



## Speed of Sound

**P26.** (a) In Helium we have  $971.9 \text{ m s}^{-1}$  (b) In Deuterium we have  $890 \text{ m s}^{-1}$

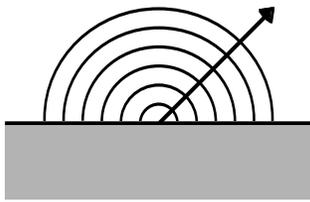
From the difference, we conclude (correctly) that the speed of sound depends the structure of gas molecules, and not just their mass. Just in case you are wondering this is pretty amazing: we can deduce facts about the structure of molecules from something as gross as the speed of sound.

**P27.** (a) Dry air at STP  $331.45 \text{ m s}^{-1}$  at STP

(b) Dry air at  $20^\circ\text{C}$   $343.4 \text{ m s}^{-1}$  at  $20^\circ\text{C}$

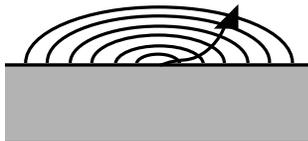
**P28.** The lightning flash is observed essentially instantaneously. The thunder is emitted at the same time as the lightning flash, but travels outward from the explosion at roughly  $330 \text{ m s}^{-1}$ . (See previous question). So the sound travels 1 km every 3 seconds. Thus if the sound is heard a time  $t$  after the lightning flash then it has travelled a distance  $\approx t/3$  kilometres where  $t$  is in seconds

**P29.**



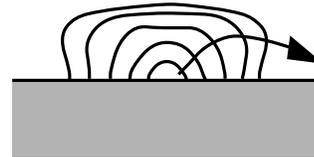
### Uniform Temperature

The arrow shows the direction of travel of sound i.e. perpendicular to the wavefronts



### Normal Case,

Normally it gets colder with increasing altitude. The arrow shows the direction of travel of sound i.e. perpendicular to the wavefronts



### Temperature Inversion

Now it gets warmer with increasing altitude. The arrow shows the direction of travel of sound i.e. perpendicular to the wavefronts

**P30.** This is one of those interesting questions that has many interesting answers, none of which are definitive. Give yourself marks if you mentioned any of the following:

- There are still plenty of molecules around even at a thousandth of atmospheric pressure there are typically  $10^{22}$  molecules per cubic metre.
- Although the speed of a sound wave does not depend explicitly on pressure, the *energy* tied up in the oscillating sound wave is proportional to the amount of gas oscillating which *is* proportional to pressure.
- As the air is removed from the chamber, the loudspeaker still emits sound waves, but they carry less and less energy
- This can be described as an impedance matching problem
- Eventually a regime is reached where the separation between molecules is of the same order as the wavelength of sound. At this point the amplitude of the sound wave is effectively zero. Another way to think of it is to think that at this point all sound waves are effectively random density fluctuations.

**P31.** About  $61 \times 10^3$  seconds ( $\approx 17$  hours) for sound waves travelling at  $330 \text{ m s}^{-1}$  for a distance of  $2\pi \times 6.4 \times 10^6 / 2 = 20 \times 10^6$  metres through the atmosphere

About 3200 seconds ( $\approx 1$  hour) for seismic waves (sound waves) travelling at  $\approx 4000 \text{ m s}^{-1}$  for a distance of  $2 \times 6.4 \times 10^6 = 12.8 \times 10^6$  metres directly through the Earth

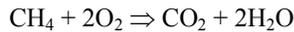
**P32.** Yes it can be built and is used. I believe aircraft use an ultrasonic device based on this idea to measure the air temperature. Problems might involve (a) impurities in the air (e.g. moisture content) (b) distinguishing changes in the speed of sound due to changes in  $\gamma$  and changes in temperature (c) changes in the dimensions of the apparatus with temperature.

The typical dimension over which the sound wave is measured should be either (a) of the order of a wavelength of sound if a resonant technique is used, or (b) much greater than a wavelength if a time of flight technique is used. Since  $f \approx 1 \text{ kHz}$  corresponds to  $\lambda \approx 0.3 \text{ m}$ , then the frequency used should operate at  $f \gg 1 \text{ kHz}$  if the apparatus is not to be too large. An ultrasonic device ( $f \approx 20 \text{ kHz}$ ) would be smaller and easier on the ears too!

### E33. Flame

The inputs to a burner are general air ( $\text{O}_2, \text{N}_2$ ) and some kind of hydrocarbon ( $\text{CH}_4, \text{C}_2\text{H}_6, \dots$ ).

The exothermic reaction that drives the flame process is generally of the form



However, most of the gas volume ( $\approx 80\%$ ) entering the burner is actually  $\text{N}_2$  and so the flame contains  $\text{N}_2$  together with  $\text{N}_2\text{O}, \text{NO}, \text{NO}_2$  (generically called NOX) which is generated in the hot post-reaction region of the flame. Furthermore, if the reaction is not quite balanced (e.g. fuel rich or fuel poor) the flame region will also contain either excess hydrocarbon or excess oxygen. If there is excess fuel then the full reaction to  $\text{CO}_2$  may be inhibited and  $\text{CO}$  formed instead

So the post flame region can contain:

Gas	M	Gamma ( $\gamma$ )	Comments
$\text{CH}_4$	16	1.31 @ ? °C	Temperature of measurement not known
$\text{CO}$	28	1.3 @ 1800 °C	
$\text{CO}_2$	44	1.2 @ 500 °C	
$\text{O}_2$	32	1.4 @ 20 °C	Gamma will fall above around 1500 K (see Figure 5.5) heading to roughly 1.3
$\text{N}_2$	28	1.4 @ 20 °C	
$\text{H}_2\text{O}$	18	1.334 @ 100 °C	
$\text{N}_2\text{O}$	44	1.324 @ ? °C	Temperature of measurement not known

So it's a fair guess that in the flame region the average value of  $\gamma$  is in the range  $1.3 \pm 0.05$

There is much more uncertainty over the average value of molecular mass (that determines the density). Here we have to guess that nitrogen is still the main component of the gas. Water is lighter than nitrogen and carbon dioxide is heavier than nitrogen so if they will tend to compensate for each other a little. A fair guess is that the effective molecular mass will lie in the range  $28 \pm 3$ .

$$c_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$$

So the best guess for the flame temperature is  $T = \frac{c_{\text{sound}}^2 M}{\gamma R}$

The uncertainty in the flame temperature will be given by roughly:

$$\left[\frac{\Delta T}{T}\right]^2 \approx \left[\frac{\Delta M}{M}\right]^2 + \left[\frac{\Delta \gamma}{\gamma}\right]^2$$

$$\left[\frac{\Delta T}{T}\right]^2 \approx \left[\frac{3}{28}\right]^2 + \left[\frac{0.05}{1.3}\right]^2$$

$$\left[\frac{\Delta T}{T}\right]^2 \approx 0.107^2 + 0.0385^2$$

$$\left[\frac{\Delta T}{T}\right] \approx 0.11$$

I have assumed that there is essentially no uncertainty in determining the speed of sound and it certainly is much less than 10%. This analysis predicts that at 2000 °C (2273 K) the uncertainty of temperature would be about 250 K (not great!) With a bit more understanding of the combustion process one could probably narrow down the estimate of molecular mass a little more cleverly and actually achieve something of the order of 5% in temperature.

Thanks to Gavin Sutton for advice on this question

**P34.** You should find  $\Delta T/T = (1 - 1/\gamma)(\Delta P/P)$  evaluates to  $\Delta T \approx 0.17$  mK.

$$\frac{\Delta T}{T} = \left[1 - \frac{1}{\gamma}\right] \frac{\Delta P}{P}$$

$$\Delta T = \left[1 - \frac{1}{1.4}\right] \frac{0.2}{10^5} 293$$

$$= 1.67 \times 10^{-4} \text{ K}$$

$$\approx 0.17 \text{ mK}$$

These oscillations would be extremely hard to measure. Try to use an extremely low mass detector

- a micron thin foil of platinum held on low thermal conductivity leads?
- A low mass thermocouple with very thin wires
- A micro sensor chemically machined out of silicon with a thermistor integrated into it

**P35.** We estimate  $v_{\text{RMS}} = \left[\frac{3}{\gamma}\right] c_{\text{sound}}$  i.e.  $v_{\text{RMS}} = \left[\frac{3}{1.4}\right] 337 = 493 \text{ ms}^{-1}$

The agreement is pretty good. According the Maxwell speed distribution we should find average speed  $\approx 454 \text{ ms}^{-1}$  and an RMS speed  $493.2 \text{ ms}^{-1}$ . Its clear that the speed of sound really is a good predictor of the RMS speed.

**P36.** Basically any speed of sound technique could be used and it would be very difficult you separate out the  $T^{1/2}$  dependence of the speed of sound from the variations of  $\gamma$  with temperature. I guess one would use a noble guess as the test gas. Care must be taken to ensure that all the apparatus is at the same temperature. A clever device made at NPL measured the changes in the resonant frequencies of a spherical cavity and operated as a kind of thermometer. Through positive feedback it is possible to force the device to continuously resonate and so the frequency (which can be measured with high precision) will change with temperature.

**P37.** Perhaps surprisingly one needs to go to ultra high frequencies. For a given pressure amplitude, the temperature oscillations have a fixed magnitude (Question P34 above). At very low frequencies, the hot and cold regions have more time to equilibrate, but the wavelength is lengthened by an exactly compensating amount. At higher frequencies, the same compensation holds true: The hot and cold regions are closer together, but they have less time to equilibrate. However at some point the wavelength will become of the order the mean free path, and at this point... well I don't know what would happen. If the mean free path is 30 nm and this corresponds to half a wavelength, then the frequency would be:

$$f = \frac{c_{\text{sound}}}{\lambda}$$

$$\approx \frac{300}{60 \times 10^{-9}}$$

$$\approx 5 \times 10^9 \text{ Hz}$$

$$(5 \text{ GHz})$$

### Electrical Properties

**P38.**

$$C = \frac{\epsilon \epsilon_0 A}{d}$$

$$= \frac{1 \times 8.85 \times 10^{-12} \times 10^{-4}}{10^{-3}}$$

$$= 0.885 \times 10^{-12} \text{ F}$$

$$= 0.885 \text{ pF}$$

(a) air at *STP*  $5.36 \times 10^{-2} \%$

(b) CO<sub>2</sub> at *STP*  $9.88 \times 10^{-2} \%$

(c) CO<sub>2</sub> at 1 atmosphere and 100 °C  $7.23 \times 10^{-2} \%$

(d) CO<sub>2</sub> at 100 atmospheres and 100 °C.

The answer can be read from Figure 5.23 as  $\approx 10 \%$ .

**P39.**

gas	$\epsilon$	$\alpha(\text{F m}^2)$
He	1.000065	$2.14 \times 10^{-41}$
Ne	1.00013	$4.28 \times 10^{-41}$
Ar	1.000516	$16.98 \times 10^{-41}$
N <sub>2</sub>	1.000516	$18.00 \times 10^{-41}$

Helium is the least polarisable, and Nitrogen the most, which is related to ease of distorting the charge cloud around each molecule. In turn this is related to the relative magnitudes of (a) the internal electric field generated by the other electrons and the nucleus, and (b) the externally applied field.

**P40.**  $\epsilon = 1.0060$   $p_p = 4.73 \times 10^{-30} \text{ C m}$  (or 4.73 debye units)

**P41.** Correction factor of 1.02 leads to  $3.19 \times 10^6 \text{ V m}^{-1}$

**P42.** High pressure operation is required. Maximum conceivable breakdown field has a high  $u_o$  (say  $\approx 10 \text{ eV}$ ) and a mean free path of one atomic diameter  $\approx 0.3 \text{ nm}$ . yields  $\approx E_{\text{breakdown}} \approx 3000 \text{ MV m}^{-1}$  (much larger than can be achieved in practice)

## Optical Properties

**P43.**

	$n_{\text{light}}$	$c_{\text{light}} (\text{ms}^{-1})$
Vacuum	1	$2.9979 \times 10^8$
Air	1.000293	$2.9970 \times 10^8$
He	1.000036	$2.9978 \times 10^8$
Xe	1.000702	$2.9958 \times 10^8$

**P44.** Using  $a = 0.391 \text{ nm}$  and  $\lambda = 0.4 \text{ }\mu\text{m}$  gives a ratio of 1023 (i.e.  $\approx 1000$ )

**P45.** If  $(\epsilon - 1) = \text{constant} \times P$  then:

$$\begin{aligned} \epsilon - 1 &= \text{constant} \times P \\ \epsilon &= 1 + \text{constant} \times P \\ n_{\text{light}} &= \sqrt{\epsilon} \\ &= \sqrt{1 + \text{constant} \times P} \\ &= [1 + \text{constant} \times P]^{\frac{1}{2}} \\ &\approx 1 + \frac{1}{2} \text{constant} \times P + \dots \end{aligned}$$

**P46.**

- (a) Vacuum  $5.8898 \times 10^{-7} \text{ m}$   
 (b) Air at *STP*  $5.8881 \times 10^{-7} \text{ m}$   
 (c) Air at  $0 \text{ }^\circ\text{C}$  and  $1.023 \times 10^5 \text{ Pa}$   $5.8880 \times 10^{-7} \text{ m}$

The direction of propagation of a wave is perpendicular to its wave fronts and the distortion of the wave fronts redirects the wave like a randomly fluctuating, very long focal length lens. It is analogous to the shadowy patterns seen on the bottom of a swimming pool when there are waves on the surface.

**P47.** Assuming the curvature of the Earth can be neglected and that the angular change is roughly the same as if all the refraction occurred at a single interface, one can use Snell's Law

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_{\text{light}}(\text{medium } 2)}{n_{\text{light}}(\text{medium } 1)}$$

to find:

$$\begin{aligned}
\sin \theta_2 &= \sin \theta_1 \frac{n_{light}(vacuum)}{n_{light}(atmosphere)} \\
&= \sin 45^\circ \frac{1.000000}{1.000293} \\
&= \frac{1}{1.000293\sqrt{2}} \\
&= 44.983^\circ
\end{aligned}$$

This is the angle with the vertical and so the angle with horizon will be raised slightly to  $45.016^\circ$ .

**P48.** Figure 3.8 shows the use of an optical interferometer to determine the refractive index of light in a gas. Initially the sample arm of the apparatus is at vacuum. For a path length in the gas of 50 cm and yellow light with a frequency of  $5.09 \times 10^{14}$  Hz, calculate the pressure of helium gas required to produce a single wavelength phase shift between the two beams.

**P49.** On a clear day one can see a very long way, I guess  $\lambda \approx 50$  km. On a hazy summer day this could easily be reduced to  $\lambda \approx 1$  km.

**P50.** Neon has massive near IR activity which overcomes the insensitivity of the eye in this region. In fact the eye has very long tail of near IR sensitivity. Look at a spectrum and see how much of it is red — the answer is most of it!

**P51.** Number density of liquid  $\approx 3.346 \times 10^{28} \text{ m}^{-3}$  : Number density of vapour  $\approx 2.687 \times 10^{25} \text{ m}^{-3}$

Ratio  $\approx 1245$  so expect that the difference of the refractive index of the vapour from unity scales by this factor i.e.  $\approx (1.33 - 1)/1245 = 265 \times 10^{-4}$  i.e. expect to find refractive index is 1.000265

Table 5.18 gives a value of 1.000254: Amazing!