

Chapter 2 Answers

These are answers to the exercises in Chapter 2 of:

Understanding the Properties of Matter
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

Thanks

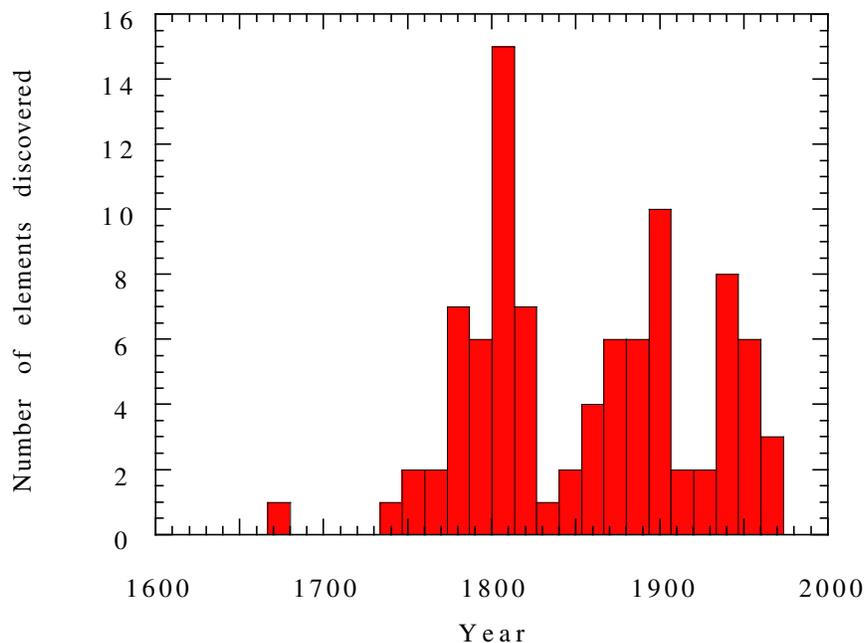
Michael@physicsofmatter.com

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P1. Hydrogen is the most common element in the outer part of the sun. Gold is $\approx 8 \times 10^{-6}$ parts per million, PPM i.e. ≈ 8 parts in 10^{12} or 0.008 parts per billion, PPB)

P2. Gold really is rare on Earth! The concentration of copper is $\approx 10^2$ PPM while gold is only present at $\approx 10^{-3}$ PPM. i.e. there is roughly 10^5 times less gold than copper. Interestingly, in my experience, the prices of copper and gold differ only by a factor between 10^2 and 10^3 depending on preparation.

C3.



Basically, new techniques of isolation became available. The peak in the early part of the last century was (I think) due to use of electrolysis to isolate metals, and the peak in the latter part of the century was due to the use of spectroscopy. The peak in the 20th century is (I think) from the heavier radioactive elements which is due to the development of nuclear chemistry.

P4. My best is very poor: Any suggestions would be welcome!

Ruby Strolled Yelling Z(r)ealously while Nabob Moaned Technically at Ruth. Ralph & Paddy Argued over Cadmium: InSane!

Its almost easier just to remember the elements!

P6. Following Example 2.1 we find: (a) $F \approx -2.095 \times 10^{-8}$ N at 0.105 nm (b) $F \approx -2.560 \times 10^{-8}$ N at 0.095 nm. so ΔF is $(2.560 \times 10^{-8} - 2.095 \times 10^{-8}) \approx 0.464 \times 10^{-8}$ N for an extension of 0.01 nm i.e. the force constant is of the order:

$$\frac{0.464 \times 10^{-8} \text{ N}}{0.1 \times 10^{-9} \text{ m}} \approx 460 \text{ N m}^{-1}$$

Hence, we find that an electron would oscillate at:

$$\begin{aligned} f_o &\approx \frac{1}{2\pi} \sqrt{\frac{K}{m_e}} \\ &\approx \frac{1}{2\pi} \sqrt{\frac{460 \text{ N m}^{-1}}{9.1 \times 10^{-31} \text{ kg}}} \\ f_o &= 3.58 \times 10^{15} \text{ Hz} \end{aligned}$$

and would radiate light with a wavelength:

$$\begin{aligned} \lambda_o &= \frac{c}{f_o} \\ &= \frac{3 \times 10^8}{3.58 \times 10^{15}} \\ &= 8.38 \times 10^{-8} \\ &\approx 1 \times 10^{-7} \text{ m} \end{aligned}$$

which is in the ultra Violet region of the electromagnetic spectrum

From Example 7.26 we find $\lambda_0 \approx 0.099 \times 10^{-6}$ m and from Figure 7.52 we find experimental values for glasses of the order of 0.1 μm . i.e. all these numbers are of the same order of magnitude.

Comment

Now you might think that this is not a case in which simple harmonic might be expected. And you would be right in pointing out that this calculation is not quite kosher. This is because it is using a classical calculation in a regime where it is really not valid. However, the slightly vague reasoning goes like this:

- There is wave function for an electron around a proton which minimises its *total* energy. In this position, the attractive force of the electron to the nucleus is compensated for by the increased kinetic energy required to occupy a wave function close to the nucleus. This constitutes an effective pressure preventing the collapse of the wave function to a point.
- In this equilibrium position the kinetic and potential energy of an electron are typically of the same order. At this point we can imagine the radial force on an electron to be zero.
- If we increase or decrease the radius away from this equilibrium value then the balance of the forces acting will act to return the electron to its equilibrium separation.
- In this calculation we get nothing more than a feel for the order of magnitude of the forces involved. The important point to note is that:
 - The force constants are very ‘ordinary’ in their magnitude because is the origin of ordinary force constants in springs and bent bits of metal and so forth.
 - The frequencies of oscillation of electrons are such that the electromagnetic radiation associated with their motion is in the UV region of the spectrum.

P6. From Equation 2.17 the refractive index is $n_{\text{light}} = 2$ and the dielectric constant is: $\epsilon = 4$

P7. Since the energy of a photon is given by:

$$E = hf$$

and the X-radiation has a frequency 10^4 times greater than the optical radiation., then the molecule would need to absorb 10^4 photons of optical radiation to absorb the same energy.

P8. Since:

$$f_o = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

we have:

$$\begin{aligned} K &= 4\pi^2 f_o^2 m \\ K &= 4\pi^2 (1.13 \times 10^{13})_o^2 \times 14u \\ &= 117 \text{ Nm}^{-1} \end{aligned}$$

when we use the mass of a single atom mass.

$$\begin{aligned} K &= 4\pi^2 f_o^2 m \\ K &= 4\pi^2 (1.13 \times 10^{13})_o^2 \times \left[\frac{1}{14u} + \frac{1}{14u} \right]^{-1} \\ K &= 4\pi^2 (1.13 \times 10^{13})_o^2 \times 7u \\ &= 58.6 \text{ Nm}^{-1} \end{aligned}$$

using the reduced mass, assuming that the atoms have equal mass.

P9. Taking the ratio of the two amplitudes we find:

$$\frac{A_1}{A_2} = 0.5$$

Substituting from Equation 2.29 we find:

$$\frac{A_1}{A_2} = \frac{F_0 / m \sqrt{(\omega_0^2 - \omega^2)^2 + 4(\alpha_2/2m)^2 \omega^2}}{F_0 / m \sqrt{(\omega_0^2 - \omega^2)^2 + 4(\alpha_1/2m)^2 \omega^2}}$$

$$\frac{1}{2} = \frac{(\omega_0^2 - \omega^2)^2 + 4(\alpha_2/2m)^2 \omega^2}{(\omega_0^2 - \omega^2)^2 + 4(\alpha_1/2m)^2 \omega^2}$$

We can then simplify this when $\omega = \omega_0$ as follows:

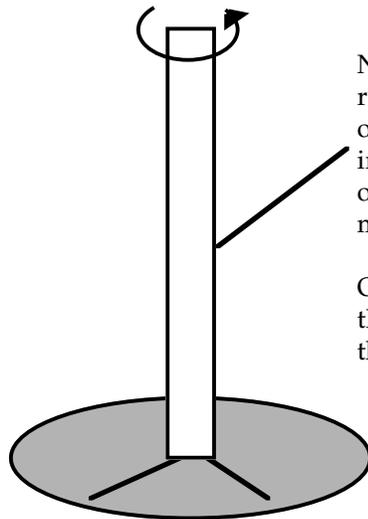
$$\frac{1}{2} = \frac{(\alpha_2/2m)^2}{(\alpha_1/2m)^2}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{1}{\sqrt{2}}$$

The damping constants differ by a factor $2^{0.5}$ i.e. 1.414

P10.

Broadly one implements some form of forced oscillator and measures the amplitude at or close the resonant frequency. My best guess for a practical device would be as illustrated right.



Need to arrange that this rod is **not** too rigid. i.e. one applies fixed amplitude oscillations to the top end and immerses the lower end in the liquid or gas. In some situations the rod might be fine wire rather than a rod.

One would determine the amplitude of the oscillations by observing marks on the rotating disk.

C11. See example animations on the web site under the heading of EQUATIONS for Chapter 2

P12. The minimum energy is given by:

$$\frac{3h^2}{8mL^2}$$

when $n_x = n_y = n_z = 1$

Electron

(i) (a) electron in box $L = 0.1\text{nm}$ yields $E_{\min} = 1.81 \times 10^{-17}\text{J} \approx 113\text{ eV}$

(i) (b) electron in box $L = 1\text{nm}$ yields $E_{\min} = 1.81 \times 10^{-19}\text{J} \approx 1.13\text{ eV}$

i.e. an electron in a 'box' of size a fraction of a nanometre has a zero-point energy of the same order as the coulomb binding energy of the electron in the atom. This is not a coincidence

Proton

(ii) (a) proton in box $L = 0.1\text{nm}$ yields $E_{\min} = 9.85 \times 10^{-21} \approx 0.061\text{ eV}$

(ii) (b) proton in box $L = 1\text{nm}$ yields $E_{\min} = 9.85 \times 10^{-23}\text{J} \approx 6.15 \times 10^{-4}\text{ eV}$

i.e. an proton in a 'box' of size a fraction of a nanometre has a small zero-point energy compared with an electron. When confined to box of nuclear dimensions ($\approx 10^{-14}\text{ m}$) then the zero point energy is $\approx 6\text{ MeV}$, which is of the same order as the strong binding energy of a proton in a nucleus. This is not a coincidence either.

Note that a hydrogen atom is essentially a proton and within solids hydrogen atoms are tremendously mobile, moving through the crystal lattice formed by other atoms and ions. There are generally a large number of places within the crystal lattice in which a proton can be trapped with a binding energy similar to that which you just calculated. In general, the proton can be freed to diffuse if the temperature is raised such that $k_B T > \text{binding energy}$ which typically corresponds to a few hundred kelvin. Often hydrogen atoms are still actively diffusing within solids at room temperature.

P13. We find:

$$E_{\min} = 0.5 hf_0 = 0.5h \frac{1}{2\pi} \sqrt{\frac{K}{m}} = 0.5\hbar \sqrt{\frac{K}{m}}$$

(a) electron in harmonic potential yields

$$f_0 = 3.46 \times 10^{15}\text{ Hz}$$

$$E_{\min} = 1.15 \times 10^{-18}\text{ J} \approx 7.16\text{ eV}$$

(b) proton in harmonic potential yields

$$f_0 = 8.07 \times 10^{13}\text{ Hz}$$

$$E_{\min} = 2.64 \times 10^{-20}\text{ J} \approx 0.167\text{ eV}$$

As we saw in question P5 electron resonant frequencies correspond to electromagnetic radiation in the UV region of the spectrum. The hydrogen atom (proton) trapped in this simple harmonic potential vibrates at a frequency in the infra red region of the electromagnetic spectrum.

P14. There are 6 states with energy $E = 14 \times \frac{h^2}{8mL^2}$

$$(3,2,1) (3,1,2) (2,3,1) (2,1,3) (1,2,3) (1,3,2)$$

C15. There is really no point to my putting my code for how to do this on the web given the variety of languages that one can solve the problem in. But the general algorithm goes something like this:

- First, notice that (in units of $h^2/8mL$) the energy range to be searched is from

$$3 \leq [n_x^2 + n_y^2 + n_z^2] \leq 40$$

Since n_i has a minimum value of 1 the maximum value of n_i that needs to be considered is

$$1 \leq [n_i^2] \leq 38$$

$$1 \leq |n_i| \leq \sqrt{38} \approx 7$$

- Next we create an array with which to create the histogram called (say) H with 40 elements: In BASIC the code for this looks like this:

```
Dim H(40) AS INTEGER
```

- So now we need to consider all the values of n_x, n_y, n_z in the range 1 to 7. In BASIC the code for this looks like this:

```
FOR nx = 1 TO 7
  FOR ny = 1 TO 7
    FOR nz = 1 TO 7

      ' CODE

    NEXT nz
  NEXT ny
NEXT nx
```

- Where 'CODE' is placed in the above code snippet you need to write lines to:
 - Evaluate the energy of the state
 - If the energy of the state considered comes to say E (in units of $h^2/8mL$) then you add one to H(E)

$$H(E) = H(E) + 1$$

- When all values of n_x, n_y, n_z have been considered, you can either write the code to plot a histogram yourself, or else export the data to a graph plotting to make a pretty histogram.

Note that there are several strategies you can adopt to speed up the process by avoiding the counting of similar states e.g. (2,1,1) and (1,2,1) but modern computers are so fast it is not worth the effort to code for these possibilities and then possibly make a mistake.

If you have enjoyed this calculation, try repeating the calculation for increasing energy ranges. You should notice that the calculation slows down quite quickly. In Chapter 6 we spend some time coping with the fact that to describe a metal we need to count (roughly speaking) the first 10^{23} quantum states. As I state in the text, if I had taken the trouble to count *every* clock cycle on *every* computer that has *ever* existed, then I would arrive at a number of the order of 10^{23} or 10^{24} so direct counting of quantum states is not a possibility.

P16. We follow Example 2.6 precisely:

(a) Strictly $\Delta N = \int_{E_1}^{E_2} g(E) dE$ but as $g(E)$ varies little between E_1 and E_2 then one may use:

$$\begin{aligned}\Delta N &\approx g(E) \int_{E_1}^{E_2} dE \\ &= g(E) \Delta E\end{aligned}$$

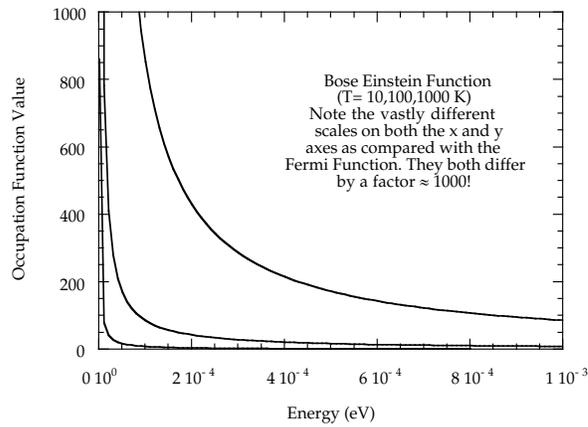
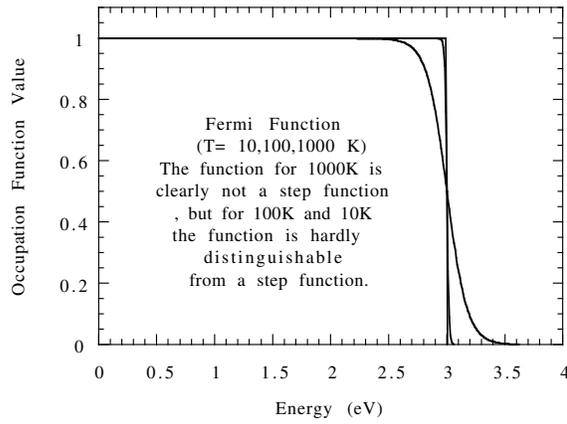
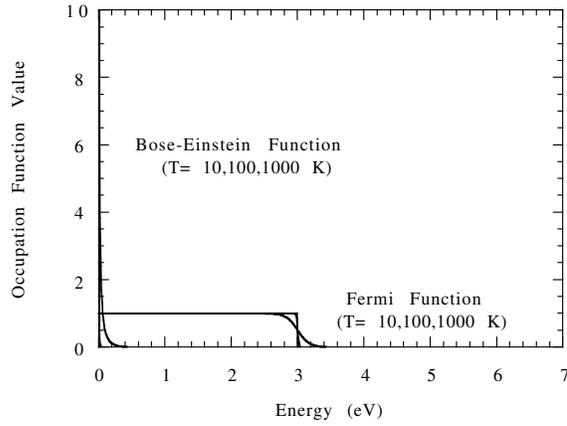
so

$$\begin{aligned}\Delta N &= [A\sqrt{E}] \Delta E \\ &= [A\sqrt{4}] 0.01 \\ &= 0.02A \text{ states}\end{aligned}$$

(b) Now we have to consider a wide range of energies across which the occupation function varies considerably: BY the definition of the density of states function:

$$\begin{aligned}\Delta N &= \int_{E_1}^{E_2} A\sqrt{E} dE \\ &= A \left[\frac{2}{3} E^{3/2} \right]_0^{4.5} \text{ states} \\ &= 6.36A \text{ states}\end{aligned}$$

C17 & C18 See example animations on the web site under the heading of EQUATIONS for Chapter 2. The graphs below may also help.



C20. (a) If there are thought to be around 10^{87} protons in the Universe then their mass is around

$$\text{Mass} = 10^{87} \times 1.66 \times 10^{-27} = 1.66 \times 10^{60} \text{ kg}$$

$$\text{The volume would be } 1.66 \times 10^{60} \text{ kg} / 10^4 \text{ kg m}^{-3} = 1.66 \times 10^{56} \text{ m}^3$$

(b) The Earth has a volume $4\pi r^3/3 = 1.1 \times 10^{21} \text{ m}^3$ which is a factor 10^{35} or so smaller than the volume of the condensed Universe.

If this was assembled into a sphere its radius would be given by $4\pi r^3/3 = 1.66 \times 10^{56} \text{ m}^3$ which evaluates to $3.4 \times 10^{18} \text{ m}$ or $3.4 \times 10^{15} \text{ km}$. For reference the distance from the Sun to the most distant planet in our solar system Pluto is roughly $6 \times 10^{12} \text{ m}$ and 1 light year is around $9.4 \times 10^{15} \text{ m}$. So the radius of the sphere would be about 360 light years!

C21. Simply I must wait

$$t = 6.02 \times 10^{23} / 400 \times 10^6 = 1.505 \times 10^{15} \text{ seconds}$$

Which amounts to around 47 million years and the warranty will indeed have expired.. To put this in perspective, *homo sapiens sapiens* the species to which you (probably) belong evolved around 2 to 3 million years ago; the Earth is thought to have come into existence around 4000 million years ago and the universe is thought to be around 15000 million years old.

It is difficult to estimate even roughly the number of clock cycles executed by *every* computer that that has *ever* existed. Around 30 years ago there were essentially no computers. Now I guess there are probably around 1000 million PCs in existence, most of them quite old. The fastest have come into existence in the last 10 years. If we give each one a putative clock speed of 100 MHz and assume it has been operating for the last 10 years continuously we find

$$N = 10^9 \times 10^8 \times [3600 \times 24 \times 365] = 3.15 \times 10^{24} \text{ clock cycles}$$

Depending on the assumptions you made, you should find answers of the order of the Avogadro number, or probably slightly greater.