

Chapter 7 Answers

These are answers to the exercises in Chapter 7 of:

Understanding the Properties of Matter
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

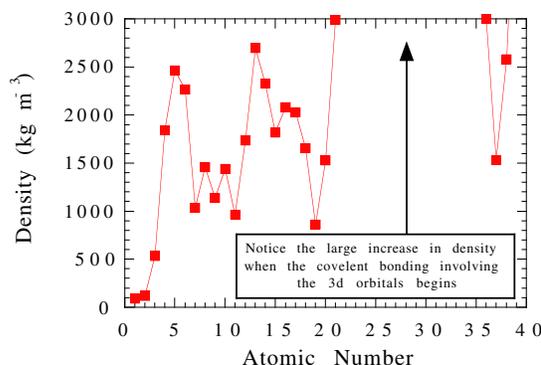
Thanks

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Density

P1. The ‘shoulders’ on the high atomic number edges of peaks *e*, *f* and *g* correspond to filling of *3d*, *4d*, and *5d* electron sub-shells respectively.

C2. Yes, the periodicity is still there.



P3. *Osmium* and *Iridium*. These are not the elements with the greatest *number density* of atoms. It is *possible* for a compound or alloy of Osmium to be denser than Osmium itself, but I do not know of any. I would expect the disorder introduced in an alloy in general to reduce the effect of any minor reduction in bond length. One possibility might be to alloy very small atoms (e.g. H) which would sit interstitially between the atoms of Os.

C4 *Beryllium* is the element with the greatest *number density* of atoms.

P5.

	W	Al	units
Density	19254	2698	kg m ⁻³
Number Density	6.307×10^{28}	6.024×10^{28}	m ⁻³
Molar Density	0.105×10^6	0.100×10^6	mol m ⁻³
Molar Volume	9.5×10^{-6} (i.e. ≈ 9.5 cc)	10.0×10^{-6} (i.e. ≈ 10.0 cc)	m ³ mol ⁻¹
Atomic Volume	1.586×10^{-29}	1.66×10^{-29}	m ³
Typical Separation	2.51×10^{-10}	2.55×10^{-10}	m

P6. *Ebony* will sink in water. *Ebony* and *Teak* will sink in ethanol

P7. No! It implies nothing other than that the *actual* density of tin (which depends on its crystal structure [which is similar to that of diamond in Example 7.3] and nearest neighbour separation) *by chance* happens to give the same density as our simple model.

Compressibility

P8. By considering the correlation between compressibility and density data, estimate the compressibility of Uranium ($Z = 92$)(Figure 7.4).

P9. Which two elements listed in Table 7.XX have the highest values of bulk modulus. For these elements, estimate roughly the pressure required to *reduce* the average distance between atoms by 1%. Compare the pressure change required with the temperature change required to increase of average distance between atoms by 1% ?

Thermal expansivity

P10. (a) Cadmium and Zinc both have $\approx 30 \times 10^{-6} \text{ K}^{-1}$ (b) Lead is next with $\approx 29 \times 10^{-6} \text{ K}^{-1}$

P11. Values of thermal expansivity are $\alpha_{\text{Brick}} \approx 7 \times 10^{-6} \text{ K}^{-1}$ and $\alpha_{\text{Cement}} \approx 12 \times 10^{-6} \text{ K}^{-1}$, so estimate the effective expansivity for Brick and Cement $\alpha_{\text{House}} \approx 10 \times 10^{-6} \text{ K}^{-1}$. This makes calculations easy, but the correct answer is probably closer to brick as most houses have \approx a greater thickness of brick than cement.

(a) Change in mean height of house = *height* $\times \alpha_{\text{House}} \Delta T = 10 \times 10 \times 10^{-6} \times 10 = 10^{-3} \text{ m} = 1 \text{ mm}$

(b) As in (a) the answer is 0.22 mm. Remember that the space where the door would have been expands as if it were filled with brick and cement.

P12. Basically we follow the argument given in Example 7.4

A circle has initial area $A = \pi r^2$. Radius expands $r \longrightarrow r + \Delta r$ and so the new area is:

$$A + \Delta A = \pi(r + \Delta r)^2$$

$$A + \Delta A = \pi r^2 + 2\pi r \Delta r + \underbrace{\Delta r^2}_{\text{neglect}}$$

$$\frac{\Delta A}{A} = \frac{2\pi r \Delta r}{\pi r^2}$$
$$= \frac{2\Delta r}{r}$$

$$\frac{\Delta A}{A} = 2\underbrace{\alpha}_{\Delta T} \Delta T$$

P13. As above

P14. From Table 11.1 one can find a typical melting temperature ($\approx 1000 \text{ K}$ to 2000 K) and using a typical value for α (10^{-5}) one can estimate the typical bond length increases by $\approx 1\%$ to 2% . In fact it will be rather larger than this since the α tends to increase with temperature. It is interesting to compare this with the amplitude of atomic vibration at melting of about 5% of the separation between atoms (Lindemann theory of melting, §11.2) we can see that atomic vibrations are highly asymmetric at the melting point.

P15. At $-50 \text{ }^\circ\text{C}$ the diameter of the hole is:

$$= 10.000(1 + 11 \times 10^{-6}[-70])$$

$$= 10.000(1 - 7.7 \times 10^{-4})$$

$$= 9.9923 \text{ mm}$$

So this is also the diameter of the copper sphere at $-50 \text{ }^\circ\text{C}$. So at $+20 \text{ }^\circ\text{C}$ the sphere will have diameter:

$$\begin{aligned}
&= 9.9923(1 + 16.7 \times 10^{-6} [+70]) \\
&= 9.9923(1 + 11.69 \times 10^{-4}) \\
&= 10.00398 \text{ mm} \\
&\approx 10.004 \text{ mm}
\end{aligned}$$

The staff in the workshop at the National Physical Laboratory can make components that fit only at low temperatures and which are then held together at room temperature. This is known as an *interference fit*.

P16. From Equation 7.14 we write $\Delta L = L_0 \alpha \Delta T$ and so the fractional change is:

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

This applies to any length within a copper sample including the *average* separation of two atoms. We use $\alpha = 16.7 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ from Table 7.7 to find:

(a) From 20 °C to 100 °C we find $16.7 \times 10^{-6} [100 - 20] = 1.34 \times 10^{-3} = 0.134\%$

(b) From 20 °C to 1000 °C we find $16.7 \times 10^{-6} [1000 - 20] = 0.0164 = 1.64\%$

(c) From 20 °C to 10 K we find $16.7 \times 10^{-6} [(-263) - (+20)] = 16.7 \times 10^{-6} [-283] = -4.73 \times 10^{-3} = -0.473\%$

All the answers (a) to (c) are likely to be inaccurate because α changes with temperature. (a) is about right because the temperature range is not too great; (b) is an underestimate because α increases with temperature; (c) overestimates the magnitude of the change because α decreases at low temperatures and in fact become very small below the temperature of liquid nitrogen (77 K)

P17, P18, P19. I wrote the book so you have to write the explanations, but the key points are listed below:

P17. Alloys

- Element consists of just a single type of atom
- Alloy consists of an atom by atom mixture of elements. In practice ideal alloys are rarely formed. Real alloys contain regions either rich or poor in the constituent elements and all manner of crystalline defects
- Thermal expansion arises from an increase in the *average* interatomic separation. In elements A and B there are only A-A and B-B bonds respectively. In an alloy of A and B, there are also A-B bonds. The increase in the *average* interatomic separation of A-B bonds with temperature may be different from either A-A and B-B bonds

P18. Invar

- Invar is an alloy of iron and nickel with approximately twice as many iron atoms as nickel atoms (36%Ni/64%Fe)
- Around room temperature invar has an anomalously low thermal expansion, around 10 times smaller than other constructional metals.
- Given the discussion in P17, we see that the effect must arise from A-B type bonds i.e. Fe-Ni bonds. Iron and nickel are both ferromagnetic and the ‘magnetic’ interactions between iron and nickel atoms have a strong dependence on the separation between the atoms. This gives rise to a temperature

range in which the *average* length of Fe-Ni bonds gets *shorter* as the temperature increases. For invar alloys this decrease closely cancels the increase in *average* length of Fe-Fe and Ni-Ni bonds.

- The word ‘magnetic’ in the above is used carefully. As you will learn in web chapter 2, the interactions between atoms which give rise to the unusual magnetic behaviour of nickel and iron are in fact entirely electrostatic in origin!

P19. Plastics

- The interactions between molecules in plastics are rather weak. If the molecules were shorter (but with the same molecular weight) then the substances would probably be liquids or gases at room temperature.
- Plastics are solids at room temperature because the *polymer* molecules from which they are made are massively entangled with each other.
- In the language of the Gibbs Free Energy function (Chapter 10), the main contribution to the free energy comes from the entropy S rather than the internal energy U .
- The thermal expansion in crystalline solids is due to increased amplitude of vibration in asymmetric potential.
- The thermal expansion in plastics is due to increased amplitude of vibration of the massive polymer chains, but the chains are still bound into the solid by their entanglement even though the interactions between polymer chains are relatively weak leading to large amplitudes of vibration.

Question: Does that mean if I looked closely at a piece of plastic I would see millions of writhing entangled snake-like chains vibrating intensely?

Answer: Yes

Speed of sound

P20. From Table 7.9

- | | | | | | |
|---------------|-----------------------|------------|-----------------------|------------------|-----------------------|
| (a) pyrex | 5640 ms ⁻¹ | (b) ice | 3840 ms ⁻¹ | (c) polyethylene | 2000 ms ⁻¹ |
| (d) aluminium | 6374 ms ⁻¹ | (e) copper | 4759 ms ⁻¹ | (f) lead | 2160 ms ⁻¹ |

P21. A typical value for the Poisson ratio is $\sigma \approx 2$. The Poisson ratio can be found from Equation 7.29 as follows:

$$R = \frac{c_L}{c_T} = \sqrt{\frac{2(1-\sigma)}{1-2\sigma}}$$

$$R^2 = \frac{2(1-\sigma)}{1-2\sigma}$$

$$R^2(1-2\sigma) = 2(1-\sigma)$$

$$R^2 - 2\sigma R^2 = 2 - 2\sigma$$

$$2\sigma(1-R^2) = 2 - R^2$$

$$\sigma = \frac{2 - R^2}{2(1 - R^2)}$$

(a) Titanium $c_L/c_T = 1.93$ $\sigma = 0.317$

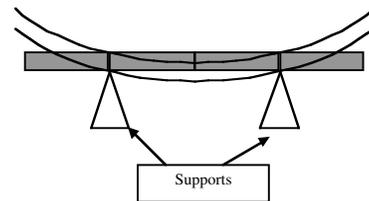
(b) Beryllium $c_L/c_T = 1.46$ $\sigma = 0.058$

(c) Lead $c_L/c_T = 3.08$ $\sigma = 0.44$

Please note error in first edition specification c_T for Lead: should be 700 ms^{-1} and not 7000 ms^{-1}

P22. This is an amazingly complicated question!

My guess is $c \approx 4500 \text{ ms}^{-1}$ which predicts that at 440 Hz the bar will be $\lambda/2 \approx 5 \text{ m}$ long. This is not correct as a glance at any kind of xylophone will reveal. The reason is that the waves which propagate on a xylophone are *flexural* waves that are not described by any of the formula that I have given in the book. The basic mode of flexing is shown on the right. It is very easy to make waves of this kind and the speed of sound is correspondingly low which is why xylophones are not gargantuan instruments.



A careful inspection of a xylophone will reveal that two bars which sound an octave apart (i.e. one has twice the frequency of the other) have a length ratio of $1:\sqrt{2}$ and not 1:2 as might be expected. This is because the speed of these waves depends on the wavelength of the wave. You can appreciate this fairly readily if you consider a thick plank of wood suspended as illustrated above. Long wavelengths are relatively easy to create compared with short wavelength where the plank just does not appear to flex at all. In fact the relationship between frequency and wavelength for these waves is $c = f\lambda^2$ so that the speed of sound is given by:

$$\underbrace{c_{\text{sound}}}_{c/\lambda} = f\lambda$$

$$c_{\text{sound}} = c/\lambda$$

So the speed becomes very low when the wavelength is very long.

P23. The definitions are in the book in §7.5.3! But basically, shear deformations *bend* bonds rather directly compressing them.

P24.

	c_L	c_T	c_L/c_T	ρ	σ	$G(\text{GPa})$	$E(\text{GPa})$	$B(\text{GPa})$	$B(\text{GPa})$ Experimental from Table 7.4
Cu	4759	2325	2.05	8933	0.34	48.2	129	134.4	137.8
Ag	3704	1698	2.18	10500	0.367	30.3	82.8	103.8	100.7
Au	3240	1200	2.7	19281	0.42	27.8	79.0	164.6	217

The Bulk modulus is inferred from Equation A2.33

$$B = \frac{E}{3(1-2\sigma)} \quad (\text{A2.33})$$

P25. Hit a long metal object hard while a companion some distance away puts their ear to the metal object. An outdoor pipe or fence is probably the best kind of object. If the object is ≈ 30 metres long there should be a delay of order of 0.1 second between hearing the sound through the air and through the object. The longer the object the better. The general formula is that the difference in time of propagation over a distance L of sound through air and solid is:

$$\Delta t = L \left[\frac{1}{c_{\text{air}}} - \frac{1}{c_{\text{solid}}} \right]$$

Heat Capacity

P26. (a) gold $25.42 \text{ JK}^{-1}\text{mol}^{-1}$ (b) neodymium $27.45 \text{ JK}^{-1}\text{mol}^{-1}$

P27. Molar heat capacity: Highest is Bromine ($75.69 \text{ JK}^{-1}\text{mol}^{-1}$) which is a liquid at room temperature. The highest solid is Gadolinium ($37.03 \text{ JK}^{-1}\text{mol}^{-1}$). The lowest is diamond (C) $8.527 \text{ JK}^{-1}\text{mol}^{-1}$

C28. Molar specific capacity: Highest is Americium $3540 \text{ JK}^{-1}\text{kg}^{-1}$ Lowest is Lithium $106 \text{ JK}^{-1}\text{kg}^{-1}$. The general formula is derived below. Remember one mole = $A \times 10^{-3} \text{ kg}$ where A is the relative molecular mass of the element.

Heat Capacity of 1 mole C_p

Heat Capacity of $A \times 10^{-3} \text{ kg}$ C_p

Heat Capacity of 1 kg $C_p/(A \times 10^{-3})$ which is the specific heat capacity

P29. At 77 K we can estimate from the graph that:

Copper $\approx 12.5 \text{ JK}^{-1}\text{mol}^{-1}$

Silver $\approx 18 \text{ JK}^{-1}\text{mol}^{-1}$

Gold $\approx 19.5 \text{ JK}^{-1}\text{mol}^{-1}$

E30. The first thing to realise that integral of the heat capacity curve in Figure 7.19 is the thermal internal energy. This is either the thermal energy which must be added to heat copper at $T = 0 \text{ K}$ to 300 K , or the thermal energy which must be removed to cool copper at $T = 300 \text{ K}$ to heat it to 0 K ($\approx 4 \text{ K}$)

Guessing an average value for the heat capacity of copper in the range 0 K to 300 K as $\approx 18 \text{ J mol}^{-1} \text{ K}^{-1}$, we ‘guess-integrate’ the area under the graph to yield $\approx 18 \times 300 = 5400 \text{ J mol}^{-1}$ and hence the simple answer is $5400/2000$ which is 2.7 liquid litres of helium per mole of copper. A mole of copper is 64 g and 1 kg requires $2.7/(64 \times 10^{-3}) \approx 42$ litres which would cost $\approx \text{£}130$. An expensive business.

In fact, one uses the cooling power of the cold gas in addition to its latent heat. The average heat content (enthalpy) of one mole of *gas* is $\approx (\Delta T/2) \times C_p(\text{He}) \approx 3120$ joules and one liquid litre contains ≈ 30 moles. Thus the ‘cooling power of one liquid litre is 2000 J of latent heat, plus $30 \times 3120 = 93600$ J of ‘heat capacity’ (or enthalpy). i.e. the latent heat is only a small fraction ($\approx 1/50$) of the available cooling power. The actual cost to cool the helium is thus reduced (in the most favourable circumstances) to $\approx \text{£}2.50$ which is much more acceptable.

P31. The Einstein temperature is clearly lower for gold than for copper. My guess is $\Theta_E \approx 110 \text{ K}$ which (following Example 7.13) implies $f_0 \approx 2.29 \times 10^{12} \text{ Hz}$ which implies $K \approx 67.8 \text{ Nm}^{-1}$.

P30. If we apply a force F between two planes of material a distance a apart then the planes increase their separation by Δa . If the planes have area A , then there are $[A/a^2]$ springs each with spring constant K . and so assuming the force equally spread across all the springs*, the effective spring constant between the planes is $[A/a^2] \times K$. So in terms of this effective spring constant we can write that the extension Δa is given by:

$$\Delta a = \frac{F}{[\text{effective spring constant}]}$$

$$\Delta a = \frac{F}{[AK/a^2]}$$

From the definition of Young’s modulus (Equation 7.20) We have

$$\frac{F}{A} = E \frac{\Delta a}{a}$$

$$E = \frac{F/A}{\Delta a/a}$$

$$E = \frac{F/A}{\frac{F}{[AK/a^2]}/a}$$

Cancelling we find: $E = \frac{K}{a}$

From the heat capacity we find : Using $a \approx 0.3 \text{ nm}$ and $K \approx 95.6 \text{ Nm}^{-1}$ yields $E = 318 \text{ GPa}$. Using Equation A2.33 $B = \frac{E}{3(1-2\sigma)}$ we can estimate $B \approx E$ for materials with a Young’s modulus close to $1/3$.

From Table 7.4 the bulk modulus is 137 GPa which is not very close to 318 GPa. So what? Well notice two things. Firstly the numbers are of the same order of magnitude (factor ≈ 2.5) and that’s pretty good for a *Mickey Mouse*TM theory such as the one developed in this question. We haven’t even considered the crystal structure which determines the real numbers of ‘springs per unit area’ which could easily contain factors of the order of 2. But I think it also reflects that the physics of the Einstein theory is just not quite

right and systematically overestimates the frequency of vibration of atoms.

*Not true for real materials and the origin of the reduction in strength of many real materials.

P33. Using $\lambda_{\min} \approx 2a$ yields

$$\begin{aligned}\Theta_D &\approx \left[\frac{h}{k_B} \times \frac{1}{\lambda_{\min}} \right] c_{\text{sound}} \\ &\approx \left[\frac{h}{k_B} \times \frac{1}{2a} \right] c_{\text{sound}} \\ &\approx \left[\frac{6.63 \times 10^{-34}}{1.38 \times 10^{-23}} \times \frac{1}{2 \times 0.3 \times 10^{-9}} \right] c_{\text{sound}} \\ &\approx [0.08] c_{\text{sound}} \\ &\approx \left[\frac{1}{12.5} \right] c_{\text{sound}}\end{aligned}$$

a value of ≈ 12.5 compared with the experimental value of approximately 7. This is pretty good agreement considering the hand waving nature of the arguments and the roughly factor 2 difference between c_T and c_L .

Using c_T for niobium (2092 ms^{-1}) this implies a Debye temperature of $\approx 299 \text{ K}$ which may be compared with the experimental value estimated from low temperature heat capacity data of 275 K .

P34. Very roughly the answers are (a) 1% accurate at $T/\Theta_D \approx 0.09$ and (b) 10% accurate at $T/\Theta_D \approx 0.12$

So the rule of thumb is: don't use the T^3 rule above $T/\Theta_D \approx 0.1$

P35. See Figure 7.27

P36. Using $E_F \approx 7.06 \text{ eV}$ and $k_B = 1.38 \times 10^{-23} / 1.6 \times 10^{-19} = 0.863 \times 10^{-5} \text{ eV K}^{-1}$: we find the ratio is:

$$\frac{8.625 \times 10^{-5} T}{7.06} = 1.22 \times 10^{-4} \text{ at } 10 \text{ K rising to } 1.22 \times 10^{-2} \text{ (i.e. } \approx 1\%) \text{ at } 1000 \text{ K i.e. always quite small}$$

E37. This is basically a straight forward question, but when you try to answer it you can easily get pretty bogged down in the complexities of it. The answer is $C_{el}/C_{\text{lattice}}$ for silver is (a) ≈ 0.0077 at 300 K and (b) 3.67 at 1 K and follows pretty closely Example 7.14

The molar electronic heat capacity is given by Equation 7.69

$$\gamma = \frac{\pi^2}{3} g_m(E_F) k_B^2 \text{ and the density of states at the Fermi Energy is given by Equation 6.70}$$

$$g_m(E_F) = \frac{V_m \sqrt{2m^3 E_F}}{\pi^2 \hbar^3} \text{ where } E_F = \frac{\hbar^2 k_F^2}{2m} \text{ and } k_F = (3n\pi^2)^{1/3}. \text{ Plugging all these together we find:}$$

$$\begin{aligned} \gamma &= \frac{\pi^2}{3} \left[\frac{V_m \sqrt{2m^3 \hbar^2 k_F^2}}{\pi^2 \hbar^3 \sqrt{2m}} \right] k_B^2 \\ &= \frac{\pi^2 k_B^2 V_m m k_F}{3\pi^2 \hbar^2} \\ &= \frac{k_B^2 V_m m (3n\pi^2)^{1/3}}{3\hbar^2} \end{aligned}$$

To estimate these we need to find the molar volume (=molar mass/density) and the number density of 'free' electrons n . For a metal with an effective valence of 1 such as silver this is the same as the number density of atoms. This is just $N_A / (\text{molar volume})$. For silver the molar mass is 0.1079 kg and the density is 10500 kg m^{-3} . Plugging it all together we arrive (after careful checking) at:

$$\text{Molar volume} = 1.0276 \times 10^{-5} \text{ m}^3 \text{ and number density} = 5.861 \times 10^{28} \text{ m}^{-3}$$

$$\begin{aligned} \gamma &= \frac{[1.38 \times 10^{-23}]^2 [1.0276 \times 10^{-5}] [9.1 \times 10^{-31}] [3 [5.861 \times 10^{28}] \pi^2]^{1/3}}{3 [1.054 \times 10^{-34}]^2} \\ &= [5.343 \times 10^{-14}] [3 [5.861 \times 10^{28}] \pi^2]^{1/3} \\ &= 6.42 \times 10^{-4} \text{ J K}^{-2} \text{ mol}^{-1} \end{aligned}$$

$$\text{So the electronic heat capacity is given by: } C_{el} = \gamma T = 6.42 \times 10^{-4} T \text{ J K}^{-1} \text{ mol}^{-1}$$

We can see from Figure 7.19 that the lattice heat capacity at 300 K is around $25 \text{ J K}^{-1} \text{ mol}^{-1}$ and so we can answer part (a) of the question. The answer is:

$$\frac{C_{el}}{C_{lattice}} = \frac{6.42 \times 10^{-4} \times 300}{25} \approx 7.7 \times 10^{-3}$$

or just less than 1%.

At low temperatures the lattice heat capacity is given by Equation 7.61 with $\Theta_D = 225 \text{ K}$ from Table 7.12. So we find:

$$C_{lattice} \approx \frac{1944}{225^3} T^3 \approx 1.75 \times 10^{-4} T^3 \text{ J K}^{-1} \text{ mol}^{-1}$$

So at 1 K we find the answer to part (b) to be:

$$\frac{C_{el}}{C_{lattice}} = \frac{6.42 \times 10^{-4} \times 1}{1.75 \times 10^{-4} [1]^3} \approx 3.67$$

i.e. at low temperatures the electronic heat capacity is greater than the lattice heat capacity.

P38. For silicon, any conduction electrons are frozen out at these temperatures. On a personal note I will add that I spent many years trying to make this not happen and it is indeed possible but due to some silly mistakes I made, I never managed it! Anyway, there are essentially no free electrons: if there were their heat capacity would easily exceed the feeble lattice heat capacity at these temperatures. We can estimate

$\Delta T = \Delta Q/C$. The heat input $\Delta Q = 3 \times 10^4 \text{ eV} = 4.8 \times 10^{-15} \text{ J}$. The heat capacity can be estimated from Equation 7.61 as:

$$C_V = \frac{1994}{630^3} T^3 = 7.97 \times 10^{-6} T^3 \text{ J K}^{-1} \text{ mol}^{-1} \text{ i.e. this is the heat capacity of 28.09 g of silicon}$$

The density of silicon is 2329 kg m^{-3} and so the mass of 1 cm^3 (10^{-6} m^3) is $2.329 \times 10^{-3} \text{ kg}$. So the heat capacity of 1 cm^3 is:

$$\begin{aligned} C &= 7.97 \times 10^{-6} T^3 \text{ J K}^{-1} \text{ for } 28.01 \text{ g} \\ C &= 7.97 \times 10^{-6} T^3 \text{ J K}^{-1} \text{ for } 28.01 \times 10^{-3} \text{ kg} \\ C &= \frac{7.97 \times 10^{-6} T^3}{28.01 \times 10^{-3}} \text{ J K}^{-1} \text{ for } 1 \text{ kg} \\ C &= \frac{7.97 \times 10^{-6} T^3}{28.01 \times 10^{-3}} \times 2.329 \times 10^{-3} \text{ J K}^{-1} \text{ for } 1 \text{ cm}^3 \end{aligned}$$

which amounts to $6.6 \times 10^{-7} T^3 \text{ J K}^{-1}$. So we can evaluate $\Delta T = 4.8 \times 10^{-15} \text{ J} / 6.6 \times 10^{-7} T^3 \text{ J K}^{-1}$ which evaluates to $\frac{7.24 \times 10^{-9}}{T^3}$ kelvin.

T	ΔT	$\Delta T/T$
10^{-3} K	7 K (not accurate!)	Very large
10^{-2} K	$7.2 \times 10^{-3} \text{ K}$	≈ 1
10^{-2} K	$7.2 \times 10^{-6} \text{ K}$	small

So, yes, you can make a detector this way. At the two lower temperatures there is a large fractional change in temperature. Devices such as this have been used in the search for Weakly Interacting Dark Matter. As I mentioned above, the problem with this is that there are essentially no conducting electrons and so the change in conductivity with temperature is very hard to measure.

Electrical Properties

P39. In order Best to fourth best: Silver, Gold, Copper, Aluminium.

No. Several elements have become superconducting with DC resistivity indetectably different from zero. Of these elements, only Aluminium becomes superconducting.

Amongst the lanthanide elements, Gadolinium is the worst and Ytterbium the best.

P40. (a) Copper $1.55 \times 10^{-8} \Omega \text{ m}$, (b) Brass $6.3 \times 10^{-8} \Omega \text{ m}$, and (c) Zinc $5.5 \times 10^{-8} \Omega \text{ m}$

P41. The resistivity of copper is $1.673 \times 10^{-8} \Omega \text{ m}$ and so the resistance of the wire is:

$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{[1.673 \times 10^{-8}] \times [50 \times 10^3]}{\pi(0.05)^2} \\ &= 0.1065 \Omega \end{aligned}$$

So current through the wire is $i \approx V/R$ which is $\approx [3 \times 10^5] / 0.1065 \approx 2.81 \times 10^6 \text{ A}$. The total power dissi-

pated is

$$i^2 R = [2.81 \times 10^6]^2 \times 0.1$$

$$= 789 \times 10^9 \text{ W}$$

So the power dissipated per metre is $789 \times 10^9 / [50 \times 10^3] \approx 15.78 \times 10^6 \text{ W i.}$ around 15 MW per metre. This wire would melt!

P42. In my view ρ_Z is not a reliable predictor of ρ_{Z+1} , because the curves are very ‘jumpy’. However, it can form a fair guess if the elements are “similar” and you have no other information!

Ballpark Figures:

Good Metals $10^{-8} \Omega \text{ m}$

Most Metals $10^{-7} \Omega \text{ m}$

Poor Metals $10^{-6} \Omega \text{ m}$

Overall, all elemental metals fall within a factor 10 either way of $10^{-7} \Omega \text{ m}$

P43. The assumption to make is that very approximately ρ is directly proportional to absolute temperature. So we estimate that:

$$\frac{\rho_{77}}{\rho_{293}} \approx \frac{77}{273}$$

$$\frac{\rho_{77}}{4.51 \times 10^{-8}} \approx \frac{77}{273}$$

$$\rho_{77} \approx \frac{77}{273} \times 4.51 \times 10^{-8}$$

$$\approx 1.3 \times 10^{-8} \Omega \text{ m}$$

P44. Following Example 7.17, you should find answers something like those in the tables below. A femtosecond (fs) is 10^{-15} s . We start with:

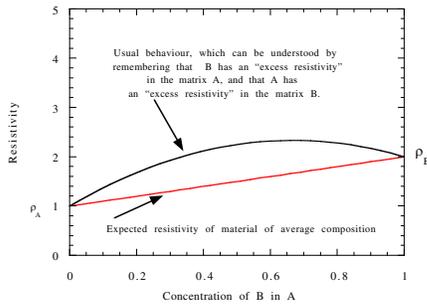
$$\Delta t = \frac{m_e \sigma}{nq^2}$$

To estimate the number density of ‘free’ electrons n we first find the *molar volume* (= molar mass/density) and then multiply an effective valence X . This is just $XN_A / (\text{molar volume})$.

	σ	Number density of atoms	X	n	Δt (fs)
Au	4.255×10^7	5.89×10^{28}	1	5.89×10^{28}	25.7
Cu	5.977×10^7	8.47×10^{28}	1	8.47×10^{28}	25.1
Zn	1.690×10^7	6.57×10^{28}	2?	13.14×10^{28}	4.6
Cu(Zn)	1.587×10^7	7.52×10^{28}	1.5?	11.3×10^{28}	5.0
Nd	0.156×10^7	2.92×10^{28}	3	8.76×10^{28}	0.6

P45. Don’t forget to send your answers to me! I’m still looking for a better answer.

P46.



P47. We start with Example 7.18:

$$C = \frac{\epsilon\epsilon_0 A}{d}$$

$$d = \frac{\epsilon\epsilon_0 A}{C}$$

$$= \frac{4.5 \times 8.85 \times 10^{-12} \times 10 \times 10^{-6}}{10^{-9}}$$

$$= 3.98 \times 10^{-7}$$

$$d \approx 0.4 \mu\text{m}$$

So from Table 7.17 we find the resistivity of quartz is around $10^{16} \Omega \text{ m}$ and so the resistance of this piece of quartz is:

$$R = \frac{\rho L}{A}$$

$$R = \frac{10^{16} \times 3.98 \times 10^{-7}}{10 \times 10^{-6}}$$

$$\approx 0.4 \times 10^{15} \Omega$$

If the capacitor has a voltage of 100 V across it the current is V/R which amounts to:

$$i = \frac{V}{R} = \frac{100}{0.4 \times 10^{15}} \approx 0.25 \text{ pA}$$

Even this thin sliver of a capacitor is an extremely good insulator. Such a device might find an application as part of an integrated circuit.

For an AC current at 1 kHz, the capacitive impedance ($2\pi f/C$) evaluates to $\approx 6.28 \times 10^{12} \Omega$ and so the Capacitive Current is $\approx 1.59 \times 10^{-11} \text{ A}$ i.e. 15.9 pA i.e. resistive current is only 1.6% of the capacitive current.

Both quartz and polystyrene are good materials with which to make a capacitor. Neither is outstandingly better than the other in these terms, and choice of material rests on other factors e.g. cost, reliability, stability etc.

P48. As in the previous question we start with Example 7.18:

$$C = \frac{\epsilon\epsilon_0 A}{d}$$

$$d = \frac{\epsilon\epsilon_0 A}{C}$$

$$= \frac{4.5 \times 8.85 \times 10^{-12} \times 10 \times 10^{-6}}{10^{-9}}$$

$$= 3.98 \times 10^{-7}$$

$$d \approx 0.4 \mu\text{m}$$

Use safety margin and assume E_{max} is between 10 MVm^{-1} and 20 MVm^{-1} (Table 7.18) and we find V_{max} is between 4 V and 8 V.

This maximum voltage would decrease strongly with increasing temperature.

Semiconductors

P49. Reading from the Figure 7.41 we estimate ρ (Si with 1 PPM phosphorus) $\approx 10^{-3} \Omega \text{ m}$ at around 300 K

Again reading from Figure 7.41 we see that this corresponds to $\approx 4 \times 10^{22}$ P atoms m^{-3} .

P50. Just something you have to remember $\Delta E \approx 1 \text{ eV}$. Although this is a typical figure, and close to the value for the archetypal semiconductor, silicon $\Delta E \approx 1.1 \text{ eV}$, you should bear in mind that band gaps vary widely from very close to zero to many eV. The range of band gaps gives materials a wide variety of interesting (and useful) properties. Very low band gap materials (Mercury Cadmium Telluride) are useful as detectors for long wavelength (λ as long as 5 microns) radiation and form parts of thermal imaging devices. Very high band gap materials (e.g. diamond) are excellent insulators but can display semiconducting behaviour at high temperatures which allows them to handle large currents and work in environmental extremes.

P51. Following Example 7.21 we first estimate the resistivity of silicon at 1000 K ($\approx 727 \text{ }^\circ\text{C}$) from Figure 7.42 as $\approx 10^{-3} \Omega \text{ m}$. We then write:

$$n \approx \frac{1}{2\rho q\mu}$$

$$\approx \frac{1}{2[10^{-3}][1.6 \times 10^{-19}][4.4 \times 10^{-3}]}$$

$$= 7.1 \times 10^{23} \text{ m}^{-3}$$

Since there of the order 10^{28} atoms of silicon per cubic metre this corresponds to the “ionisation” of around 100 atoms in every million silicon atoms.

For copper the number of conduction electrons does not change with temperature. Assuming a valence of 1, the number density of electrons in copper is the same as the number density of atoms:

$$\begin{aligned}
n &= \frac{N_A}{\text{molar volume}} \\
&= \frac{6.023 \times 10^{23}}{63.55 \times 10^{-3} / 8933} \\
&= \frac{6.023 \times 10^{23}}{7.114 \times 10^{-6}} \\
&= 8.47 \times 10^{28} \text{ m}^{-3}
\end{aligned}$$

i.e. Copper has $\approx 10^5$ times more carriers

P52. Basically just look on either side of the semiconductor in the periodic table. As ever, things in practice are not quite so simple, but it's a fair approximation.

(a) Donors for silicon: N, P, As Acceptors for silicon: B, Al, Ga

(b) Donors for germanium: P, As, Sb Acceptors for germanium: Al, Ga, In

One would expect the impurities from immediately adjacent to the semiconductor to have the lowest activation energies.

Thermal conductivity

P53. Assuming $\kappa \approx 50 \text{ W K}^{-1} \text{ m}^{-1}$ the temperature underneath of the frying pan is $\approx 201.5 \text{ }^\circ\text{C}$. This is much lower than either the flame temperature of a gas cooker ($>1000 \text{ }^\circ\text{C}$) or the glowing filaments of an electric cooker (temperature $\approx 800 \text{ }^\circ\text{C}$). But, I think its probably a fair estimate of the temperature of the metal.

Assuming the very end of the handle is at room temperature and that no heat is lost from the side of the handle (i.e. assuming it is like a heat "tube") and that $\kappa \approx 0.2 \text{ WK}^{-1} \text{ m}^{-1}$, the heat flow down the handle is 0.075 W and the temperature half way down the handle would be $110 \text{ }^\circ\text{C}$. However the heat *is lost* from the side of the handle and so the temperature would be less than this, though the exact value is hard to guess. My estimate is $\approx 40^\circ\text{C}$ which would still be a very hot object to pick up. In fact, these are the equilibrium values and in my experience if you cook with a frying pan for a long time (after the twentieth pancake for example) then the hand does become uncomfortably hot, so may be this is not such a bad guesstimate after all.

P54. For the glass we find (using the value from table 7.17)

$$\begin{aligned}
\text{heat flow} &= \frac{\kappa \times \text{Area} \times \Delta T}{\text{thickness}} \\
&= \frac{1 \times [3 \times 2] \times [23 - (-2)]}{5 \times 10^{-3}} \\
&= 30 \times 10^3 \text{ W}
\end{aligned}$$

At 30 kW this seems like an overestimate but it might explain our large gas bills! For the bricks we find

$$\begin{aligned}
 \text{heat flow} &= \frac{\kappa \times \text{Area} \times \Delta T}{\text{thickness}} \\
 &= \frac{1 \times [3 \times 2] \times [23 - (-2)]}{20 \times 10^{-2}} \\
 &= 750 \text{ W}
 \end{aligned}$$

So around 750 W flows through the wall.

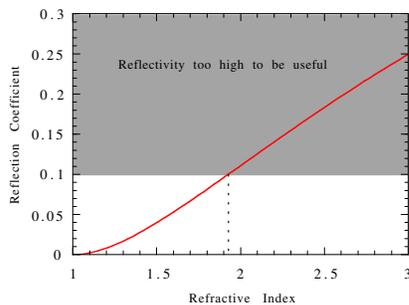
Based on this calculation, you should recommend that I invest in double-glazing. In fact my wife and I did buy double-glazed sash window from *Refurb-a-sash*, and they made a big difference. They were period styled and cost £2200, so how long before I recoup on my gas bill? At 5 pence per kWh of heating costs I need to save $2200 \times 20 \text{ kWh} = 44000 \text{ kWh}$ to break even. Assuming I work in the room for 4 hours a day and save around 1 kWh per hour that I work there I need to work there for around 11000 days or roughly 30 years! In fact, our youngest child was born within the year and we moved so I never got to really investigate the phenomenon.

Understanding how double-glazing is both simple and horribly complicated. The simple story is that first there is twice as much thickness of glass which cuts the heat loss in half. There is then around 10 mm of gas which has a typical thermal conductivity of $\approx 10^{-2} \text{ W m}^{-1} \text{ K}^{-1}$. This will probably dominate the thermal resistance of the window and cut heat losses by a factor between 10 and 100. However in detail it is extremely difficult to calculate the thermal resistance of a double glazing unit. The reasons are (a) that the conduction through the gas is dominated by convection not the still gas thermal conductivity and (b) that conduction through a poorly designed frame can easily ‘short circuit’ a highly insulating double glazed panel. Look at the NPL web site thermophysical properties section for details of how measurements are actually done. I have seen this apparatus and it is INCREDIBLE!

Optical Properties

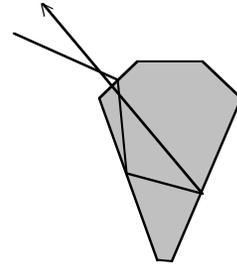
P55. From Table 7.22 we find that the average reflectivity is 92%

C56. From the graph we see that the maximum value of refractive index is ≈ 1.9



P57. No. If the prism affects all wavelengths equivalently there will be no dispersion and the white light will not be split into its component colours. A high refractive index prism would cause the white light to be deviated through a larger angle than a low refractive index prism.

P58. Diamond has an astonishingly high refractive index (≈ 2.4). Diamond is not especially dispersive and gets its unique properties for jewellery from a combination of three factors. Firstly, the crystals are cut in a clever way to facilitate the following two factors. Secondly, the high refractive index bends light strongly as it enters the crystal. Thirdly, the back facets of the crystal are cut so that light is totally internally reflected. The critical angle for total internal reflection is given by $\sin^{-1}(1/n_{\text{light}})$ which is $\approx 25^\circ$ which is much smaller than the 50° or so which is typical for normal glass. This means that a much larger fraction of light is totally internally reflected. The back facets are cut so that after two internal reflections light entering the front of the crystal is sent back out of the front of the crystal. This is what makes diamonds appear bright: you are looking at the bright lights which jewellers helpfully (cunningly?) place in their windows.



P59. Basically, that the theory has captured the essential physics of the process. And what did that essential physics amount to? Well, it was that at frequencies well below resonance (atomic transitions) the electron shells around atoms do distort linearly with applied field and obey a simple harmonic approximation to their motion. In particular this leads to the idea that the optical polarisability will increase with the square of the frequency.

P60. Following Example 7.24 we write:

$$\begin{aligned}\lambda_0 &\approx 3.34 \times 10^7 \sqrt{\frac{1.3^2 - 1}{5 \times 10^{28}}} \\ &\approx 1.24 \times 10^{-7} \\ &\approx 0.12 \text{ } \mu\text{m}\end{aligned}$$

We can estimate the resonant frequency as:

$$\begin{aligned}f_0 &= \frac{c}{\lambda_0} \approx \frac{3.0 \times 10^8}{1.24 \times 10^{-7}} \\ &\approx 2.42 \times 10^{15} \text{ Hz}\end{aligned}$$

And hence the restoring force constant is:

$$\begin{aligned}K &= m_e [2\pi f_0]^2 \\ &\approx 9.1 \times 10^{-31} [2\pi \times 2.42 \times 10^{15}]^2 \\ &\approx 210 \text{ Nm}^{-1}\end{aligned}$$

K is around half the value in Question P5 at the end of Chapter 2, but both it and f_0 are of the same order of magnitude as those estimated in Chapter 2

P61. The equation is $\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$ which (for copper at room temperature) amounts to:

$$\delta = \sqrt{\frac{2}{[4\pi \times 10^{-7}][59.77 \times 10^6][2\pi]}} \sqrt{\frac{1}{f}}$$

$$= 0.0651 \sqrt{\frac{1}{f}} \text{ m}$$

Evaluating this as required we find

- (a) @ 50 Hz $\delta = 9.2 \text{ mm}$
- (b) @ 50 MHz $\delta = 9.2 \text{ }\mu\text{m}$
- (c) @ 1 GHz $\delta = 2.1 \text{ }\mu\text{m}$
- (d) @ 10 THz $\delta = 21 \text{ nm}$
- (e) @ 1000 THz $\delta = 2.1 \text{ nm}$

The calculation in (e) indicates that at the surface of a metal, the top few nanometres, or the top few tens of atomic layers, dominate the optical reflection from the surface.

The skin depth at 50 MHz for Aluminium is $\approx 11.5 \text{ }\mu\text{m}$. S to reduce the field by a factor 100 we write:

$$0.01 = \exp(-t/\delta)$$

i.e. $t = \delta \times \ln(100) \approx 53.4 \text{ }\mu\text{m}$. The same calculation can also be use to calculate the minimum metallic thickness which will be opaque to visible light.