## Chapter 10

These are answers to the exercises in Chapter 10 of:

*Understanding the Properties of Matter*
by Michael de Podesta.

If you find an error in these answers, or think they could be clarified in any way, please feel free to contact me.

Thanks

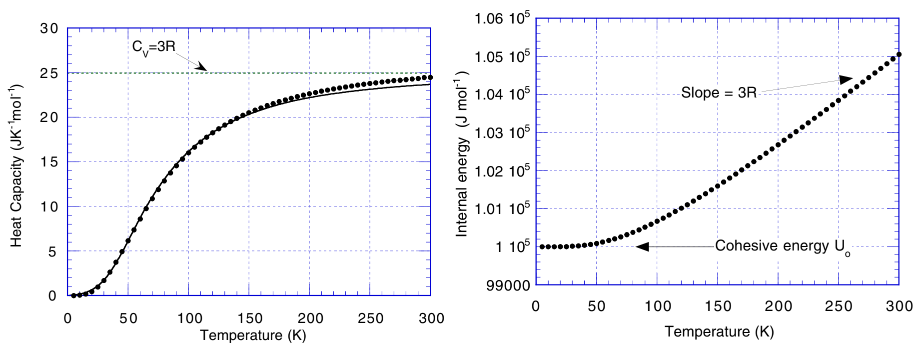
Michael@physicsofmatter.com

**P1.** In general, $U(T) = \int_0^T C(T)dT$ so $U(T) = \frac{\gamma T^2}{2} + \frac{\alpha T^4}{4}$

At 1 K $U(T) \approx 5.25 \times 10^{-5}$ J mol$^{-1}$ and at 20 K U(T) $\approx$0.42 J mol$^{-1}$ and

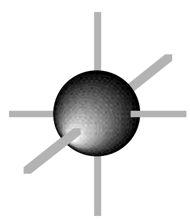
These are both very small compared to $U_0 \approx 10^5$ J mol$^{-1}$

**P2.**

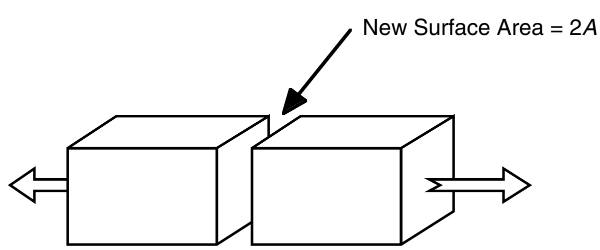



**P3.** Each atom has 6 bonds, each of which it 'half owns' i.e. associated with each atom are 6 $\times$ 0.5 = 3 bonds. So in 1 mole of substance there are $3N_A$ bonds so the energy per bond is $U_o / 3N_A$ joules

$\approx$ 0.346 eV

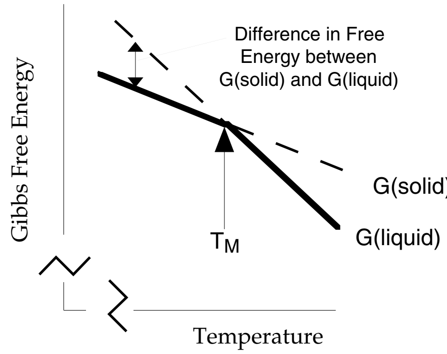


The number of bonds broken when a bar of cross sectional area $A$ is broken is $\approx A/a^2$ and each bond represents the loss of 0.346 eV of energy. The total area of surface formed is $2A$ (area $A$on each broken part) and so the energy per unit area of surface formed is $0.346 \times (A/a^2)/2A$ in eV m$^{-2}$ evaluates to 0.31 Jm$^{-2}$. In practice the surface structure changes to minimise this energy.



**P4.**

0.7 K below $T_M$ the difference in free energies is $0.7 \times (400\text{-}395) \approx 0.35\ \mathrm{JK^{-1}mol^{-1}}$




1 mole has a volume…

$N_A\, a^3 = 1.626 \times 10^{-5}\ \mathrm{m^3}$

So $\Delta G_{LS}$ per cubic metre is

$\Delta G_{LS} \approx 0.215 \times 10^6\ \mathrm{Jm^{-3}}$

Critical embryo size is $a >$

$$\frac{2\times 0.6}{0.215\times 10^6} \approx 5.6\ \mu\mathrm{m}$$

This seems unfeasible large: it involves VAST numbers of atoms all fluctuating in the same way. In practice the surface energy *difference* between the liquid and solid states is much less than the surface energy of the solid state, perhaps only ≈ few % of the 0.6 (or 0.3) $\mathrm{Jm^{-2}}$ we estimated here. This reduces the embryo size estimate to ≈ few nm, which is much more plausible.

**P5.** Following Example 10.7,

$$\left.\frac{\mathrm{d}P}{\mathrm{d}T}\right|_{\mathrm{Water}} = -15.6 \times 10^6\ \mathrm{Pa\,K^{-1}} \qquad \left.\frac{\mathrm{d}P}{\mathrm{d}T}\right|_{\mathrm{ethanol}} = +5.53 \times 10^6\ \mathrm{Pa\,K^{-1}}$$

So $T_M(\text{water}) = 273.16 - P/15.6 \times 10^6$ and $T_M(\text{ethanol}) = 155.85 + P/5.53 \times 10^6$

Equating these and solving for $P$ yields $P = 0.478$ GPa or ≈ 4.8 kBar. This is quite easily achievable, but the ice would probably have formed a new structure at around 2 kBar and the slope of the melting curve changed to a positive value (See Chapter 12 Q28). The prediction is that the melting temperatures would be the same at $T_M \approx 242.3$ K ≈ –30.9 °C. In fact the minimum melting temperature for water is around –20 °C.

**P6.** I don't know for sure whether the slipperiness is connected with the temperature of the sliding object, but I would expect it be, and I find the premelting hypothesis very convincing.

**P7.**

| | $T$ | $P$ | |
|---|---|---|---|
| (a) | 200 | $10^5$ | Solid |
| (b) | 200 | $10^8$ | Liquid according to diagram, but actuality solid (See Chapter 12 Q28) |
| (c) | 400 | $10^8$ | Liquid |
| (d) | 400 | $10^5$ | Gas |
| (e) | 700 | $10^5$ | Gas |
| (f) | 700 | $10^8$ | Liquid/Gas near critical point where the distinction is blurred. |
| (g) | 273.15 | 611 | Could be solid, liquid or gas: This is the triple point |